RISK-ADJUSTED STOCK INFORMATION FROM OPTION PRICES

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As option's payoff depends upon future stock price, option prices contain important information of their underlying stocks. In this paper, we price options with the physical measure where we can jointly estimate expected stock return and implied volatility from market prices of options. Using S&P 500 Index options, we discover the following four results. First, our result shows that investors have higher expectations of stock returns in the short-term, but lower expectations in the long-term. Second, the term structure of volatilities in our model is much flatter than the term structure of the Black-Scholes model. Third, the empirical investigation shows that a combination of our implied expected return and implied volatility with Black-Scholes implied standard deviation provides a better model than Black-Scholes implied standard deviation alone to forecast future volatility of stocks for any combination of moneyness and maturity. Finally, the implied volatility of our model can predict much better future volatility over the life of the option than the implied volatility of the Black-Scholes model, more so for short maturities of 90 days or less.

As option's payoff depends upon future stock price, option prices contain important information of their underlying stocks. For a bullish stock, the price of the call goes up and the put goes down. However, using the Black-Scholes model, we can only retrieve the volatility information, as risk preference disappears from the pricing model. In this paper, we price options with the physical measure where we can jointly estimate the expected return μ and implied volatility of the underlying stock from market prices of options.

Pricing measures are not unique. Yet the law of one price (known as no arbitrage) guarantees all pricing measures lead to a unique option price. As a result, there exists a pricing measure where μ is present and the same option price is obtained. In this paper, we choose the physical measure to price options so that we

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can jointly estimate the expected return and implied volatility of the underlying stock. The use of the physical measure in pricing assets has been the standard methodology in microeconomic theories. In fact, the earlier literature, such as Sprenkle (1961) and Samuelson (1965), in option pricing used the physical measure to price options. Our contribution is to extend those models and further derive the closed form solution to the expected return of the option as a function of the expected return of the stock.

Black and Scholes (1973) show that if the market is complete,[1] then the expected return of the stock should disappear from the valuation of the option as dynamic hedging (known as continuous rebalancing, price by no arbitrage, or risk neutral pricing) should effectively remove the dependence of the option price on the stock return. This is true, however, only if the market is truly complete in reality. In other words, if the reality were exactly described by the Black-Scholes model, it is impossible to theoretically solve for both the expected return and implied volatility of the stock. However, it has been empirically shown that the Black-Scholes model cannot explain all option prices (known as the volatility smile and volatility term structure). As a result, we can solve for these two parameters simultaneously under our model.

Except for the expected return parameter, the physical pricing measure adopted by our model makes the same assumptions of the Black-Scholes model. In particular, we assume the same stock price process as the Black-Scholes model does. This design is to assure that we have a closed form solution to our model. In theory, we could relax as many assumptions by the Black-Scholes model as possible and build a model that can explain every traded option price in the market place. However, in doing so, we shall lose the closed form solution; furthermore once we have as many parameters as the number of the traded options, the model can no longer "price" any option as all option prices are used to calculate parameters.

As a result, we need to seek balance between over-parameterization (having same number of parameters as option prices), under-parameterization (such as the Black-Scholes model), and computational feasibility (maintaining closed form solution). As we shall show in our empirical study, with two parameters (expected return and implied volatility), we find that our model can predict historical volatility much better than the Black-Scholes model. We use the term "historical volatility" for the ex-post volatility measured over the life of the option as explained in section III.C.1.

Option pricing models of Sprenkle (1961), Ayres (1963), Boness (1964), and Samuelson (1965) employed the physical measure and implicitly or explicitly assumed some form of risk-adjusted model such that the investors buy and hold the options until maturity to extract the option implied return, which then could be linked to the stock return. However, none of these models provides an adequate theoretical structure to determine the implied return values.[2] Under the risk neutral pricing

1. This is complete market in the dynamic sense, as later described carefully by Duffie and Huang (1985).
2. Galai (1978) later showed that the Boness model and the Black-Scholes model are consistent.

measure, Heston (1993b) shows that, under a log-gamma dynamic assumption for the stock price, the expected stock return will show up in the pricing formula and yet the volatility disappears. Hence, his model is not capable of jointly determining both the expected return and volatility of the stock price. Nonetheless, Heston's paper shows the possibility of retaining the expected return parameter in the model with suitable adjustments to the pricing equation.

Using the S&P 500 index call and put options, we estimate expected stock return and implied volatility with our model. We use options with various strikes at a given day and compute expected return and volatility for each time to maturity. As a result, we obtain jointly the term structure of expected return and the term structure of implied volatility of the stock. We find a downward sloping term structure of expected return that is consistent with existing studies to be reviewed in details later in the empirical section. We find that implied volatility carries more information in predicting historical volatility of the stock than the Black-Scholes implied volatility.

The remainder of this paper is organized as follows. Section I presents the risk-adjusted discrete time model that retains the stock expected return in the option pricing equation. Section II presents the data and estimation methodology. Section III discusses the empirical results of our estimation. Section IV provides the concluding remarks.

I. THE MODEL

It is well known that the Black-Scholes model can be used to compute implied volatility and not implied expected return of the underlying stock, due to the fact that no-arbitrage argument renders a preference-free model and hence contains no such parameter. In this sub-section, we demonstrate that such parameter can be rediscovered via an "equilibrium" pricing approach similar to Samuelson (1965) and Sprenkle (1961). Let the stock price follow the usual log normal process under the physical measure:

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{1}$$

where the annualized instantaneous expected return is μ and the volatility is σ. The classical economic valuation theory states that any price today must be a properly discounted future payoff:

$$C_t = E_t[M_{t,T} C_T] \tag{2}$$

where $M_{t,T}$ is the pricing kernel, also known as the marginal rate of substitution, between time t and time T. The usual risk neutral pricing theory developed by Cox and Ross (1976) performs the following change of measure:

$$
\begin{aligned}
C_t &= E_t[M_{t,T}C_T] \\
&= E_t[M_{t,T}]E_t^Q[C_T] \\
&= \begin{cases} e^{-r(T-t)}E_t^Q[C_T] & \text{if interest rate is constant} \\ P_{t,T}E_t^{F(T)}[C_T] & \text{if interest rate follows a random process} \end{cases}
\end{aligned} \tag{3}
$$

where Q represents the risk neutral measure and $F(T)$ represents the T-maturity forward measure and $P_{t,T}$ is the risk-free zero coupon bond price of $1 paid at time T.[3] In this paper, we perform the change of measure in the other direction. That is:

$$
\begin{aligned}
C_t &= E_t[M_{t,T}C_T] \\
&= E_t[C_T]E_t^X[M_{t,T}] \\
&= E_t[C_T]e^{-k(T-t)}
\end{aligned} \tag{4}
$$

where X represents the measure where the option price serves as a numeraire, and k is the annualized expected instantaneous return on this option in the physical world. We then assume that the X-measure expectation of the pricing kernel takes a form of continuous discounting. Now, we can derive our option pricing formula as:

$$
\begin{aligned}
C_t &= e^{-k(T-t)}E_t[\max\{S_T - K, 0\}] \\
&= e^{-k(T-t)}\left[\int_K^\infty S_T\phi(S_T)dS_T - K\int_K^\infty \phi(S_T)dS_T\right] \\
&= e^{(\mu-k)(T-t)}S_tN(h_1) - e^{-k(T-t)}KN(h_2)
\end{aligned} \tag{5}
$$

where t and T are the current time and maturity time of the option, and K is the strike price of the option and

$$
\begin{aligned}
h_1 &= \frac{\ln S - \ln K + \left(\mu + ½\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \\
h_2 &= h_1 - \sigma\sqrt{T-t}
\end{aligned}
$$

To derive a pricing formula that contains μ, we need the following propositions. These propositions describe how implied return and volatility can be simultaneously estimated from option prices.

Proposition 1. Assume stock price S follows a geometric Brownian motion with an annualized expected instantaneous return of μ and volatility of σ. Let a call option on the stock at any point in time t is given by $C(S,t)$ that matures at time T. Let k be the annualized expected instantaneous return on this option. Then for a small interval of time Δt, the relationship between μ and k can be given by:

3. Note that the last line of the equation is a classical result of economic valuation.

$$k = r + \beta(\mu - r) \tag{6}$$

where

$$\beta = \frac{\operatorname{cov}(r_C, r_S)}{\operatorname{var}(r_S)} \tag{7}$$

and $r_S = \Delta S / S$ and $r_C = \Delta C / C$ are two random variables representing the stock return and call option return respectively during the period Δt. And r is the annualized constant risk-free rate for the period of the option. Proposition 1 can be proved without assuming the CAPM. (*Proof: See Appendix A.1.1.*)[4]

Equation (6) holds for a small interval of time Δt. We assume the distributions of stock return r_S and option return r_C are stationary over the period of the option. This implies the annualized instantaneous expected return and variance over a small interval of time and the annualized instantaneous expected return and variance over the discrete time (from time t and time T) will be the same. This also implies β is constant over this period, which means the linear relationship between k and μ as in equation (6) is valid over the life of the option from current time t to maturity time T.[5] Since our approach will be pricing the option in a discrete setting, we approximate the β over the discrete time from t to T as:

$$\beta_{t,T,K} = \frac{\operatorname{cov}\left(\frac{C_T}{C_t}, \frac{S_T}{S_t}\right)}{\operatorname{var}\left(\frac{S_T}{S_t}\right)} = \frac{S_t}{C_t}\frac{\operatorname{cov}\left(C_T, S_T\right)}{\operatorname{var}(S_T)} \tag{7a}$$

And with the assumption of stationarity as above, equation (6) holds for the life of the option as:

$$k_{t,T} = r_{t,T} + \beta_{t,T,K}(\mu_{t,T} - r_{t,T}) \tag{6a}$$

Equation (6) with equation (7), in continuous time, and equation (6a) with equation (7a) in discrete time can also be proved using the CAPM. However, for these two equations to hold it is not necessary that the CAPM should hold. The assumptions of the CAPM are much stronger so that all return distributions are stationary; however, here we need only the stationarity of the stock and the option return to obtain these two equations.

Hence stationarity assumption of r_S and r_C is a weaker assumption than what is needed for CAPM. Further, Galai (1978) shows many similarities between the continuous time and discrete time properties of r_C that support our assumption of stationarity of distribution.

4. Appendix A.2.1 provides a similar derivation for put options.
5. This can be easily seen by integrating both side of equation (6) from t to T.

Equation (5) is obtained based on the assumption that the expected return of the option k, expected return of the stock μ, and volatility σ are constants. We have also assumed that the stock price follows a geometric Brownian motion. In discrete time, equation (5) can be written as:

$$C_{t,T,K} = e^{(\mu_{t,T} - k_{t,T})(T-t)} S_t N(h_1) - e^{-k_{t,T}(T-t)} K N(h_2) \tag{5a}$$

where t and T are the current time and maturity time of the option, and K is the strike price of the option and

$$h_1 = \frac{\ln S_t - \ln K + \left(\mu_{t,T} + ½\sigma^2_{t,T}\right)(T-t)}{\sigma_{t,T}\sqrt{T-t}}$$

$$h_2 = h_1 - \sigma_{t,T}\sqrt{T-t}$$

Combining equations (7a) and (5a), we arrive at the following proposition.

Proposition 2. The $\beta_{t,T,K}$, based on the life of the option, can be written as:

$$\beta_{t,T,K} = \frac{S_t\left[e^{\sigma^2_{t,T}(T-t)} N(h_3) - \frac{K}{S_t} e^{-\mu_{t,T}(T-t)}\left(N(h_1) - N(h_2)\right) - N(h_1)\right]}{C_t\left(e^{\sigma^2_{t,T}(T-t)} - 1\right)} \tag{8}$$

where

$$h_3 = \frac{\ln S_t - \ln K + \left(\mu_{t,T} + 1½\sigma^2_{t,T}\right)(T-t)}{\sigma_{t,T}\sqrt{T-t}}$$

(*Proof: See Appendix A.1.2.*)[6]

It should be noted that we do not use the distributional properties of the market return r_M to obtain (8). Using (8), (5a), and (6a) we can solve for the call price $C_{t,T,K}$ explicitly in terms of the known values: stock price (S_t), strike price (K), risk free rate (r), time-to-maturity ($T - t$), and two important unknown parameters: expected stock return $\mu_{t,T}$ and volatility $\sigma_{t,T}$.[7] If we observe the values of two or more call options, with same time-to-maturity with different strike prices, we can

6. Appendix A.2.2 provides the corresponding derivation for put options.

7. $\mu_{t,T}$ and $\sigma_{t,T}$ represent expected stock return (μ) and implied volatility of the stock (σ), respectively, for a specific time period, where t is the date of observation of option prices, and T is the maturity date of the options.

then simultaneously solve for $\mu_{t,T}$ and $\sigma_{t,T}$.[8] In this paper we use "sigma," "implied volatility," and $\sigma_{t,T}$ interchangeably.

II. DATA AND ESTIMATION METHODOLOGY

A. Data

To extract implied expected return from option prices, we use the end-of-day OptionMetrics data of options on S&P 500 (SPX) for the last business day of every month during January 1996–April 2006. This data file contains the end-of-day stock CUSIP, strike price, offer, bid, volume, open interest, days-to-maturity, and Black-Scholes implied volatility for each option. From this dataset, we exclude all put options and options with zero trading volume. We also exclude single option records for a particular trade date and days-to-maturity.[9]

We obtain daily levels of the index and returns from CRSP. We need the returns for historical volatility computation. To match the CRSP records with option records, we use the trade date and CUSIP of the index. In our data all S&P500 records have a common CUSIP. Merging CRSP and option data by trade date and CUSIP can be used for any stock option in general.

For the interest rates, we use the St. Louis Fed's 3-months, 6-months, 1-year, 2-year, 3-year, and 5-year Treasury Constant Maturity Rates. Assuming a step-function of interest rates, we match the days-to-maturity in the option record with its corresponding constant maturity rate. For example, if the days-to-maturity of the option is less than or equal to 3 months we use 3-months rates, and if the days-to-maturity is between 3 and 6 months, we use the 6-months rate and so on.

In this paper, the results are based on the last business day observations for each calendar month. This results in 124 months, 791 different trade date and maturities combinations (on average 6.38 maturities per month), and a total of 7,865 options (9.94 different moneyness levels per trade date and maturity combination). Taking any other day of the month produces similar results. For example, we verified our results by taking first working day, second Thursday, and third Friday of every month. The results are similar. Table 1 shows the summary statistics of all moneyness S&P500 index call option input data that are used to compute $\mu_{t,T}$ and $\sigma_{t,T}$.[10]

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8. Using options with two or more strike prices, we can find values for $\mu_{t,T}$ and $\sigma_{t,T}$ that produce option prices closest to the observed prices in the least squares sense. A similar least-squares methodology was used by Melick and Thomas (1997).

9. We need at least two option records for a specific trade date and days-to-maturity to compute $\mu_{t,T}$ and $\sigma_{t,T}$.

10. The option data also contain Black-Scholes implied volatilities adjusted for stock dividends. Using this information along with the interest rates, we can reverse compute the corresponding European option price. If the European option price thus computed is higher than the bid and ask midpoint price, then we take the bid and ask midpoint price, else we take the European price as the option price to compute $\mu_{t,T}$ and $\sigma_{t,T}$. S&P 500 options are European style and the prices should reflect as such. However, minor differences exist between the reported closing prices and the prices reversely computed from end of day implied volatilities.

Table 1. Input Data Summary Statistics of S&P500 Index Options

Days-to-maturity groups	**<= 90 Days**	**> 90 Days**	**All Maturities**
Number of observations	5602	2263	7865
Days-to-maturity Mean	48.9749	321.2702	198.0316
Avg. moneyness Mean	0.9818	0.953	0.966
Std. Dev.	0.0298	0.0709	0.0579
Min	0.8757	0.6242	0.6242
Max	1.1446	1.3697	1.3697
Median	0.9832	0.954	0.9714
Number of calls used Mean	15.648	5.2263	9.9431
Std. Dev.	7.9866	3.1247	7.8171
Min	2	2	2
Max	42	26	42
Median	15	4	7
Avg. spread Mean	0.1365	0.035	0.0809
Std. Dev.	0.1347	0.0418	0.1082
Min	0.0065	0.0008	0.0008
Max	1.0806	0.3773	1.0806
Median	0.1016	0.0229	0.0427
Avg. volume Mean	603.0293	350.6503	464.8749
Std. Dev.	526.9405	660.0032	616.0207
Min	3.5	1	1
Max	4385.5	8186.75	8186.75
Median	472.8111	191.0833	301
Total open interest Mean	143911.3017	44156.9307	89304.9267
Std. Dev.	180980.9924	50192.6798	136556.5071
Min	0	0	0
Max	1336404	280941	1336404
Median	88911.5	23619	41591

This table presents the summary statistics of all moneyness month-end S&P 500 index call options having positive trading volume based on the month-end observations for the period of January 1996- April 2006. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days to maturity is less than or equal to 90 days, then the observation is in <=90 days-to-maturity group. If days to maturity is greater than 90 days it is in > 90 days-to-maturity group. Moneyness we define as the stock price divided by the strike price. For S&P500, stock price is the level of the index. Avg. volume is the average of volume of call options used for a $\mu_{t,T}$ and $\sigma_{t,T}$ pair estimate. Avg. spread is the average of spread of call options used for a $\mu_{t,T}$ and $\sigma_{t,T}$ pair estimate. Spread is defined as (offer - bid)/call price. Call price is the midpoint of bid and offer or the European option price whichever is lower. European option price is computed from Black-Scholes implied volatility in the data. Number of calls used is the number of option records that are used to compute a $\mu_{t,T}$ and $\sigma_{t,T}$ pair.

B. Estimation of Implied Expected Stock Return & Implied Volatility

We jointly estimate the implied expected stock return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) using the risk-adjusted option pricing model described in previous section. For a given trade date for S&P500 index, we have many call options with same days-to-maturity. We use all these options records to compute implied stock return and implied volatility by a method of grid search to look for the global optima that minimizes the square error. A square error is defined as the square of the difference between the market observed option price and right hand side of the equation used to compute the option price based on the observed values.[11] Since we are searching for the entire spectrum for the global optima, we need to specify search intervals, without which we would not be able to implement the search.[12] We use the implied expected return ($\mu_{t,T}$) search range from 0.0% to 200.00% and implied volatility ($\sigma_{t,T}$) search range from 0.0% to 100.00% for the grid search. To estimate implied expected returns we need two or more records with same key value of trade date, CUSIP, and days-to-maturity. Thus, all the single records for a key value cannot be used to compute implied return and are discarded. By this method, we extract the market implied return and market volatility for different days-to-maturity based on S&P500 index option prices, and corresponding S&P500 index levels.

III. RESULTS

Using the S&P 500 monthly index option prices from January 1996 till April 2006, we estimate expected stock return ($\mu_{t,T}$) and volatility ($\sigma_{t,T}$) with our model. We use options with various strikes at a given day and compute $\mu_{t,T}$ and $\sigma_{t,T}$ for each time to maturity. As a result, we obtain jointly the term structure of $\mu_{t,T}$ and the term structure of $\sigma_{t,T}$.

Using the S&P 500 index call options of all moneyness,[13] we find the following:

- A downward sloping term structure of $\mu_{t,T}$ that is consistent with existing studies to be reviewed in details later in the empirical section.
- Much flatter term structure for $\sigma_{t,T}$ than the Black-Scholes model.
- $\sigma_{t,T}$ carries more information in predicting historical volatility than the Black-Scholes implied volatility (i.e., average implied standard deviation, or $\bar{\sigma}_{t,T}^{BS}$) based on near term options maturing in 90-days or less.[14]

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11. The observed values used on the right hand side of equation (5a) are stock price, strike price, option price, days to maturity, and interest rate.

12. Theoretically an interval of $-\infty$ to $+\infty$ is the full search interval for both $\mu_{,T}$ and $\sigma_{t,T}$. However, we would not be able to practically implement such a search for global optima given limited processing power of resources. Therefore, we choose the upper and lower bound based on the most feasible interval possible from prior experience.

13. We also perform combined call and put option testing but the results, which are similar, not shown here for consideration of space.

14. Average implied standard deviation is the arithmetic average of Black-Scholes implied standard deviation of all options with different strike prices that are used to estimate $\mu_{t,T}$ and $\sigma_{t,T}$.

- A combination of our implied expected return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) with $\overline{\sigma}_{t,T}^{BS}$ provides a better model than using $\overline{\sigma}_{t,T}^{BS}$ alone to forecast future volatility for any maturity and moneyness combination.

A. The Term Structure of $\mu_{t,T}$

Table 2 shows the descriptive statistics of implied return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) using all moneyness S&P 500 index call options. To analyze the results we classify the data into different days-to-maturity groups. Thus the options whose days-to-maturity is less than or equal to 90 days are classified into "<=90" group. The options whose days-to-maturity is greater than 90 days are classified into ">90" group. Figure 1 shows $\mu_{t,T}$ and $\sigma_{t,T}$ graphs for S&P500 index call options of all moneyness. In these tables and graph, we see a term structure of $\mu_{t,T}$. For example in Table 2 for <=90 days-to-maturity $\mu_{t,T}$ is 19.5%, whereas for >90 days to-maturity it is 9.41.[15]

The term structure of $\mu_{t,T}$ implies the expected return is impacted by the time horizon of investment. McNulty et al. (2002) study the "real cost of equity capital" using option prices. They find high expected returns in the short term and low expected returns in the long term, which is consistent with our finding. They argue that the marginal risk of an investment (the additional risk the company takes on per unit time) declines as a function of square root of time. The falling marginal risk should be reflected in the annual discount rate.[16] Our term structure of $\mu_{t,T}$ is consistent with this explanation. However, unlike our approach, their approach is heuristic and lacks the theoretical foundation. Recently Camara et al. (2009) computed the cost of equity from option prices using a specific utility function and arrived at downward sloping term structure of expected stock return similar to McNulty et al. However, Camara et al.'s (2009) approach requires an intermediate parameter that needs to be computed using options of all firms before they compute the implied expected return of any individual firm. In contrast to their approach, our approach does not compute a similar parameter, and we do not assume any explicit utility function.[17]

The data points for the term structure graphs (Figure 1) are generated by nonparametric spline interpolation using the neighborhood data points. Our approach can be used to estimate the cost of equity for any time horizon of investment.[18] One of the advantages of our approach is that the expected return of a stock can be computed without using any information of the market portfolio such as the market risk premium. This implies one does not have to define what the "market" consists

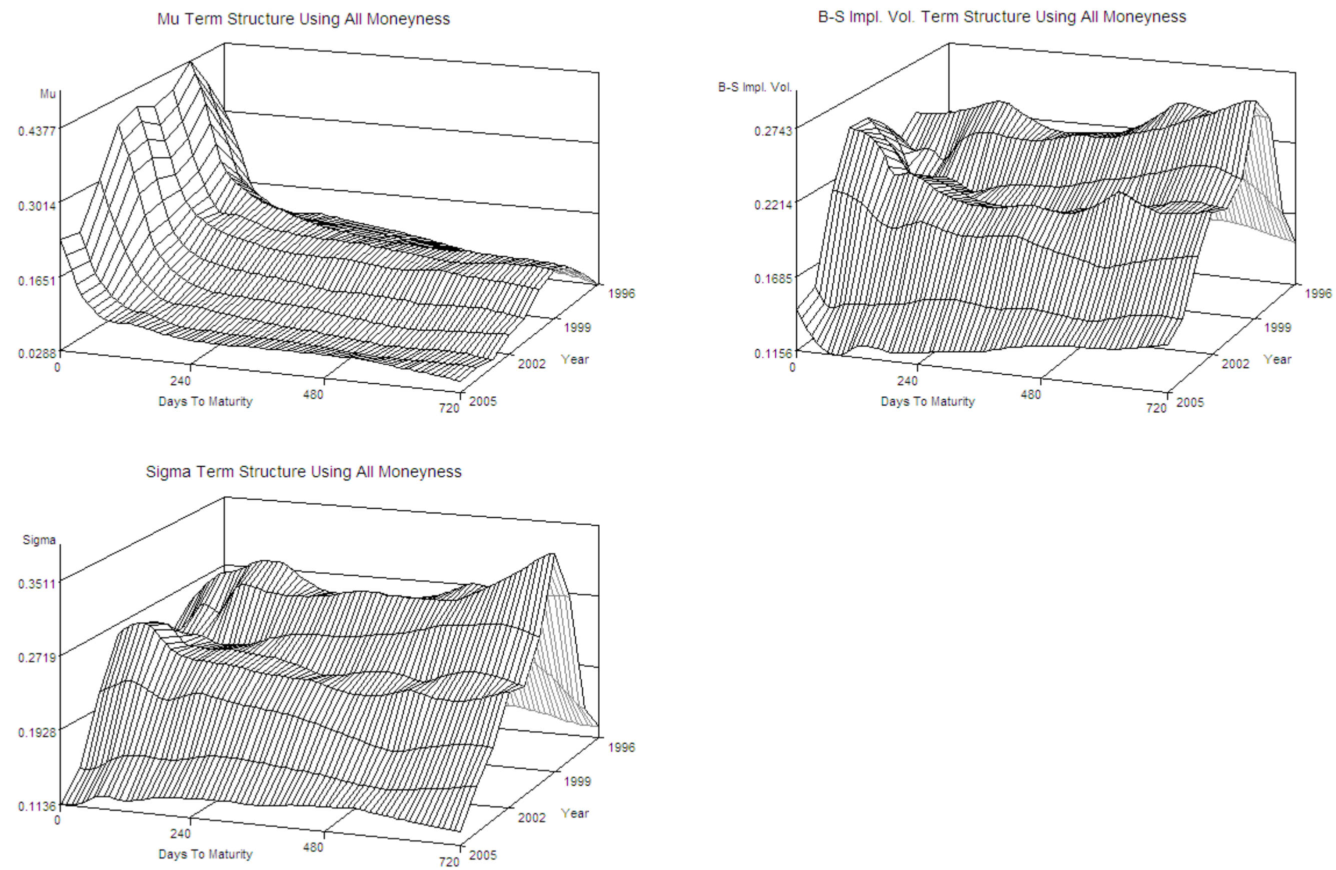
15. We also see the term structure when we group the data into 30, 60, 90, and so on days-to-maturity groups.
16. This is explained in McNulty et al. (2002).
17. Note that our model is consistent with the Black-Scholes and assumes normality of stock returns. As a result, our model is implicitly consistent with the quadratic utility function.
18. Our approach can be used to estimate cost of equity for different industry portfolios. We do similar experiments and show the term structure of expected return persists for these industry portfolios.

Table 2. Implied and Historical Summary Statistics Using S&P500 Index Options.

Days-to-maturity groups	**<= 90 Days**	**> 90 Days**	**All Maturities**
Implied expected return $\mu_{t,T}$			
Mean	0.195	0.0941	0.1397
Std. Dev.	0.0876	0.0379	0.0823
Min	0.0745	0	0
Max	0.5887	0.2428	0.5887
Median	0.173	0.0897	0.1216
Implied volatility $\sigma_{t,T}$			
Mean	0.2146	0.2132	0.2139
Std. Dev.	0.0747	0.0670	0.0705
Min	0.0788	0.1017	0.0788
Max	0.464	0.4611	0.464
Median	0.2068	0.2073	0.2071
Implied standard deviation ($\overline{\sigma}_{t,T}^{BS}$)			
Mean	0.1979	0.1942	0.1968
Std. Dev.	0.0853	0.0581	0.0785
Min	0.0738	0.0898	0.0738
Max	1.7805	1.1215	1.7805
Median	0.1854	0.1889	0.1865
Historical volatility			
Mean	0.1678	0.1712	0.1697
Std. Dev.	0.0697	0.0544	0.0618
Min	0.0632	0.0882	0.0632
Max	0.4324	0.3255	0.4324
Median	0.1553	0.181	0.166

The sample consists of all moneyness month-end S&P 500 index call options based on the month-end observations for the period of January 1996-April 2006. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in <=90 days-to-maturity group. If days to maturity is greater than 90 days it is in > 90 days-to-maturity group. We use all the call options on the same CUSIP, days-to-maturity, and trade date to compute the implied expected return and implied volatility by a grid search method that minimizes the square of difference between the observed and computed option price. Historical volatility is computed based on actual return of the index from trade date to maturity date of the option. Implied standard deviation ($\overline{\sigma}_{t,T}^{BS}$) is the Black-Scholes implied volatility. Results are in decimals.

Figure 1. Term Structures Using All Moneyness S&P500 Index Call Options.



Mu is option implied expected return, and Sigma is option implied volatility. Mu and Sigma are jointly estimated using risk adjusted model on options data.

of, and one does not have to estimate the risk premium of the market, which is required in traditional asset pricing models, to estimate the expected return.

To validate the robustness of our finding, we examine the influence of market friction proxies such as the option open interest, volume, and bid-ask spread on the term structure of implied expected return. We control for time to expiration bias, moneyness bias, and volatility bias in this regression.[19] Our results show that the market friction proxies do not explain this term structure. We also find the term structure of expected return remains for deep-in and deep-out of the money call options. Furthermore, this term structure also persists for combined call and put options (results available upon request).

B. Comparison of Term Structure of $\sigma_{t,T}$ and Black-Scholes Volatility

Our model also demonstrates a flatter (less variation) term structure of $\sigma_{t,T}$.[20] From Figure 1, we can eyeball the two volatility term structures from $\sigma_{t,T}$ of our model and $\overline{\sigma}^{BS}_{t,T}$ of the Black-Scholes model that the term structure of $\sigma_{t,T}$ is much flatter than the term structure of $\overline{\sigma}^{BS}_{t,T}$. While it is not easy to compare the two term structures statistically, we can compute the relative variation of the two term structures from Table 2. For all maturities, the mean and standard deviation of $\sigma_{t,T}$ are 0.2139 and 0.0705, respectively; and of $\overline{\sigma}^{BS}_{t,T}$ are 0.1968 and 0.0785, respectively. Hence, the relative variation, defined as standard deviation divided by the mean, is 0.3296 for our model and 0.3989 for the Black-Scholes model.[21] This demonstrates that the $\sigma_{t,T}$ of our model presents a "flatter" term structure than the $\overline{\sigma}^{BS}_{t,T}$ of the Black-Scholes model.

When we divide the sample into short term (<=90 days) and long term (>90 days), we find that our model performs better than the Black-Scholes model for the short term options to 0.3481 versus 0.4310, yet worse for the long term options to 0.3143 versus 0.2992. This demonstrates that the term structure of the Black-Scholes $\overline{\sigma}^{BS}_{t,T}$ dissipates off, for higher days to maturity options.

To have a detailed comparison of the characteristics of $\sigma_{t,T}$ of our model and the implied volatility (i.e., implied standard deviation, or $\overline{\sigma}^{BS}_{t.T}$) of the Black-Scholes model, we estimate various attributes of comparison as shown in Table 3. $\sigma_{t,T}$ is jointly estimated with $\mu_{t,T}$ using multiple option records as described in section II.A and II.B. To compute the values in this table, first, we estimate the mean and standard deviation of $\sigma_{t,T}$ and Black-Scholes implied volatility ($\overline{\sigma}^{BS}_{t,T}$) for each year and days-to-maturity based on our entire dataset. Then we compute the difference of these means and standard deviations of $\sigma_{t,T}$ and $\overline{\sigma}^{BS}_{t,T}$ for each year and days-to-maturity.[22] Panel A of Table 3 provides the summary statistics of the difference of

19. Papers by Chiras and Manaster (1978), Macbeth and Merville (1980), Rubenstein (1985), and Canina and Figlewski (1993) find these biases. Longstaff (1995) has similar controls for these biases.

20. By term structure of $\sigma_{t,T}$, we mean the value of $\sigma_{t,T}$ for different days to maturity of T, for same observation date, t.

21. Table 3 provides a detailed comparison of $\sigma_{t,T}$ and $\overline{\sigma}^{BS}_{t,T}$.

22. Difference of the means is computed as the mean of $\sigma_{t,T}$ minus the mean of $\overline{\sigma}^{BS}_{t,T}$. Similarly we compute difference of standard deviation and difference of coefficient of variation.

Table 3. Comparison of Sigma and Black-Scholes Implied Volatility.

Days-to-maturity groups	<= 90 Days	> 90 Days	All Maturities
Panel A: Test of difference between level of Sigma and $\bar{\sigma}^{BS}_{t,T}$			
Difference:			
Mean	0.0192	0.0201	0.0153
Standard Deviation	0.0232	0.0274	0.0176
t-statistics	10.9284**	13.3524**	2.88**
Panel B: Test of difference between standard deviation of Sigma and $\bar{\sigma}^{BS}_{t,T}$			
Difference:			
Mean	-0.0071	-0.0004	-0.0203
Standard Deviation	0.0313	0.0207	0.0115
t-statistics	-2.2813*	-0.1710	-5.8584**
Panel C: Test of difference between coefficient of variation of Sigma and $\bar{\sigma}^{BS}_{t,T}$			
Difference:			
Mean	-0.0565	-0.0200	-0.1232
Standard Deviation	0.1371	0.0870	0.0600
t-statistics	-4.1222**	-2.0475*	-6.8079**

This table presents the summary statistics of comparison of our sigma ($\sigma_{t,T}$) estimates and Black-Scholes implied volatility ($\bar{\sigma}^{BS}_{t,T}$) for different days-to-maturity groups based on all moneyness S&P500 Index call options for the period of January 1996-April 2006. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in <=90 days-to-maturity group. If days to maturity is greater than 90 days it is in > 90 days-to-maturity group. For this table, first, we compute mean and standard deviation of sigma and $\bar{\sigma}^{BS}_{t,T}$ for each year and days-to-maturity. Panel A presents the test of difference between mean level of sigma and for different maturity groups. Panel B presents the test of difference between standard deviation level of sigma and $\bar{\sigma}^{BS}_{t,T}$ for different maturity groups. For Panel C, we compute the coefficient of variation (CV) of sigma and $\bar{\sigma}^{BS}_{t,T}$ as corresponding standard deviation divided by the mean for each year and days-to-maturity. Then we take the difference of CV of sigma and $\bar{\sigma}^{BS}_{t,T}$ for each year and days-to-maturity. The *t*-statistics shows whether these differences are significant for different days-to-maturity groups. ** and * represent the *p*-values of less than 0.01, and between 0.01 and 0.05 respectively.

the means for different days-to-maturity groups.

Panel B provides the summary statistics of the difference of the standard deviations for different days-to-maturity groups. In Panel A, the *t*-statistics are significant for all maturity groups. Similarly in Panel B the *t*-statistics is significant for both <=90 days-to-maturity and "all maturities" groups and they are negative, showing the standard deviation is lower for sigma than $\bar{\sigma}^{BS}_{t,T}$.

Panel C shows the summary statistics of the difference of coefficient of variation (CV) of $\sigma_{t,T}$ and $\bar{\sigma}^{BS}_{t,T}$ for different days-to-maturity groups. We see the CV of $\sigma_{t,T}$ and $\bar{\sigma}^{BS}_{t,T}$ are statistically different. Similar to Table 2, we see CV of $\sigma_{t,T}$

are lower compared to CV of $\overline{\sigma}_{t,T}^{BS}$ and thus $\sigma_{t,T}$ is flatter than $\overline{\sigma}_{t,T}^{BS}$. Overall, Table 3 shows that $\sigma_{t,T}$ has lower standard deviation, lower CV, and higher mean compared to $\overline{\sigma}_{t,T}^{BS}$. This implies that $\sigma_{t,T}$ of our risk-adjusted model might have additional information beyond $\overline{\sigma}_{t,T}^{BS}$ that might be valuable to estimate the characteristics of the underlying stock.

C. Volatility Forecast

In this section we analyze whether the $\mu_{t,T}$ and $\sigma_{t,T}$ pair of our model carries more information than $\overline{\sigma}_{t,T}^{BS}$ of the Black-Scholes model in forecasting historical volatility. We use the term "historical volatility" for the ex-post volatility measured over the life of the option as explained in section III.C.1. We find that $\sigma_{t,T}$ alone can predict future historical volatility significantly better than Black-Scholes $\overline{\sigma}_{t,T}^{BS}$ when we use options of all moneyness. More interestingly, we find that when $\mu_{t,T}$, $\sigma_{t,T}$ and $\overline{\sigma}_{t,T}^{BS}$ are all used in the prediction, the result is significantly better than either $\sigma_{t,T}$ or $\overline{\sigma}_{t,T}^{BS}$ alone. These results are stronger for near term options. First, when we use all moneyness, $\sigma_{t,T}$ and its second order term do better than $\overline{\sigma}_{t,T}^{BS}$ and its second order term for both the days-to-maturity groups, namely <=90 and >90 based on adjusted R-square. Second, for near term options, the coefficients of $\sigma_{t,T}$, and the second-order term are significant even in the presence of $\overline{\sigma}_{t,T}^{BS}$. Furthermore, a likelihood ratio test rejects the null hypothesis that restricted model with $\overline{\sigma}_{t,T}^{BS}$ and its second-order term is better than the unrestricted model with all the three variables and their second-order terms for all near and far maturity groups, and for any moneyness level. [23]

A vast body of literature exists on the volatility forecasting front that investigates the forecasting capability of implied volatility from option prices.[24] In a recent comparison study, Granger and Poon (2005) find that the Black-Scholes (1973) implied volatility provides a more accurate forecast of volatilities. In their paper, they show the outcomes of 66 previous studies in this area that uses different methods to forecast volatility. These methods are historical volatility, ARCH, GARCH, Black-Scholes (1973) implied volatility, and stochastic volatility (SV).[25] Based on their ranking, they suggest that Black-Scholes (1973) implied volatility provides the best forecast of future volatility. Despite the added flexibility of SV models, the authors find no clear evidence that they provide superior volatility

23. As we show in Table 5, we take all moneyness or near-the-money options; we take <=90 days and >90 days-to-maturity groups. In all these cases we reject the restricted model that uses only Black-Scholes implied volatility and its second order term to predict the realized volatility.

24. Papers are by Latane and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981), Day and Lewis (1992), Canina and Figlewski (1993), Christensen and Prabhala (1998), Lamoureux and Lastrapes (1993), and Blair et al. (2001). Granger and Poon (2005) provide a comparison of different methods of forecasting volatility.

25. Option pricing models by Merton (1976), Cox and Ross (1976), Hull and White (1987), Scott (1987), and Heston (1993a) extend basic Black-Scholes (1973) model to incorporate stochastic volatility and jumps.

forecasts. Furthermore, they find Black-Scholes (1973) implied volatility dominates over time-series models because the market option prices fully incorporate current information and future volatility expectations. Therefore, we choose Black-Scholes implied volatility ($\bar{\sigma}_{t,T}^{BS}$) as the benchmark and compare the information content of implied expected return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) with $\bar{\sigma}_{t,T}^{BS}$. To understand the forecastability of historical volatility using $\mu_{t,T}$, $\sigma_{t,T}$ and $\bar{\sigma}_{t,T}^{BS}$, we plot these time series values in Figures 2 and Figure 3 for <=90 days-to-maturity and >90 days-to-maturity groups, respectively, for S&P500 index options using all moneyness.

1. Information Content of the Nested Model

The comparison of information content of $\bar{\sigma}_{t,T}^{BS}$ over a model of $\mu_{t,T}$, $\sigma_{t,T}$, and $\bar{\sigma}_{t,T}^{BS}$ can be evaluated using the following regressions:

$$\sigma_{t,T}^{HV} = \alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS^2} + \omega_{1t,T} \quad \text{(R1)}$$

$$\sigma_{t,T}^{HV} = \alpha_{20} + \alpha_{21}\sigma_{t,T} + \alpha_{22}\sigma_{t,T}^2 + \omega_{2t,T} \quad \text{(R2)}$$

$$\sigma_{t,T}^{HV} = \alpha_{40} + \alpha_{41}\bar{\sigma}_{t,T}^{BS} + \alpha_{42}\bar{\sigma}_{t,T}^{BS^2} + \alpha_{43}\mu_{t,T} + \alpha_{44}\mu_{t,T}^2 + \alpha_{45}\sigma_{t,T} + \alpha_{46}\sigma_{t,T}^2 + \omega_{4t,T} \quad \text{(R3)}$$

Past literature typically uses equation (R1) without the second-order term. In our investigation we include the second-order terms[26] to capture the higher order effects to explain the annualized "historical" volatility ($\sigma_{t,T}^{HV}$), where *t* is the date of observation of option prices for a given stock, and *T* is the maturity date. To compute the $\bar{\sigma}_{t,T}^{BS}$, we use the dividend adjusted Black-Scholes implied volatilities given in the OptionMetrics data file. $\bar{\sigma}_{t,T}^{BS}$ is the average of these implied volatilities of all options that are used to estimate the $\mu_{t,T}$ and $\sigma_{t,T}$ pair.[27] To compute $\sigma_{t,T}^{HV}$ first we compute daily historical volatility based on ex-post daily returns of the underlying asset for the remaining life of the option and then multiply by $\sqrt{252}$:

$$\sigma_{t,T}^{HV} = \sqrt{\frac{252}{\tau - 1}\sum_{i=1}^{\tau}\left(u_i - \bar{u}_i\right)^2}$$

where τ is the remaining life (in working days) of the option; $u_i = \ln(1 + r_i)$; r_i is the daily return of the underlying asset for day *i* in CRSP database; $\bar{u}_i$ is the mean of the u_i series.[28] Table 2 shows the summary statistics of historical volatilities ($\sigma_{t,T}^{HV}$) of S&P500 index options for different day-to-maturity groups of options.

26. We test the validity of the restricted model without the square term. Based on the likelihood ratio test, our results in most cases reject the restricted model. Therefore, we take the variables ($\mu_{t,T}$, $\sigma_{t,T}$, or Black-Scholes implied standard deviation) with the square terms.

27. $\mu_{t,T}$, $\sigma_{t,T}$ represent μ and σ respectively for a specific time period, where *t* is the date of observation of option prices, and *T* is the maturity date of the options.

28. Hull (2002) uses a similar procedure to compute historical volatilities.

Anderson et al. (2001) show that the conventional squared returns produce inaccurate forecast if daily returns are used. The inaccuracy is a result of noise in these returns. They further show that impact of noise component is reduced if high-frequency returns are used (e.g., 5-minute returns). However, a relatively recent study by Aït-Sahalia, Mykland, and Zhang (2005) demonstrates that more data does not necessarily lead to a better estimate of "realized" volatility in the presence of market microstructure noise. They show that the optimal sampling frequency is jointly determined by the magnitude of market microstructure noise and the horizon of realized volatility. For a given level of noise, the realized volatility for a longer horizon (e.g., one month or more) should be estimated with less frequent sampling than the realized volatility for a shorter horizon (e.g., one day).

Since our experiments are mostly for more than one month time horizon, the optimum data frequency should neither be 5-minutes nor the daily returns. In the absence of high-frequency data, to the extent the optimum frequency is closer to one day, our measure (of historical volatility) based on this frequency should closely represent the realized volatility.[29]

Using the above regression models, (R1) ~ (R3), we can test three hypotheses. First, we can test if $\sigma_{t,T}$ predicts better than $\overline{\sigma}_{t,T}^{BS}$. Second, we can verify if the coefficients of $\mu_{t,T}$ and $\sigma_{t,T}$ are significant even in the presence of $\overline{\sigma}_{t,T}^{BS}$. Third, we can test the hypothesis H_0: $a_{43} = a_{44} = a_{45} = a_{46} = 0$. If we reject this null hypothesis, then we can argue that $\mu_{t,T}$ and $\sigma_{t,T}$ have significant contribution in forecasting the future volatility using the model as given in equation (R3).

The regression results are shown in Table 4. We have separate regressions for different maturity groups. As before, if days-to-maturity is less than or equal to 90 days, then the observations are in <=90 days-to-maturity group. If days-to-maturity is greater than 90 days, then the observations are in >90 days-to-maturity group. We estimate these regressions using the generalized method of moments. Using OLS may not be appropriate for our data in the presence of nonspherical disturbances.

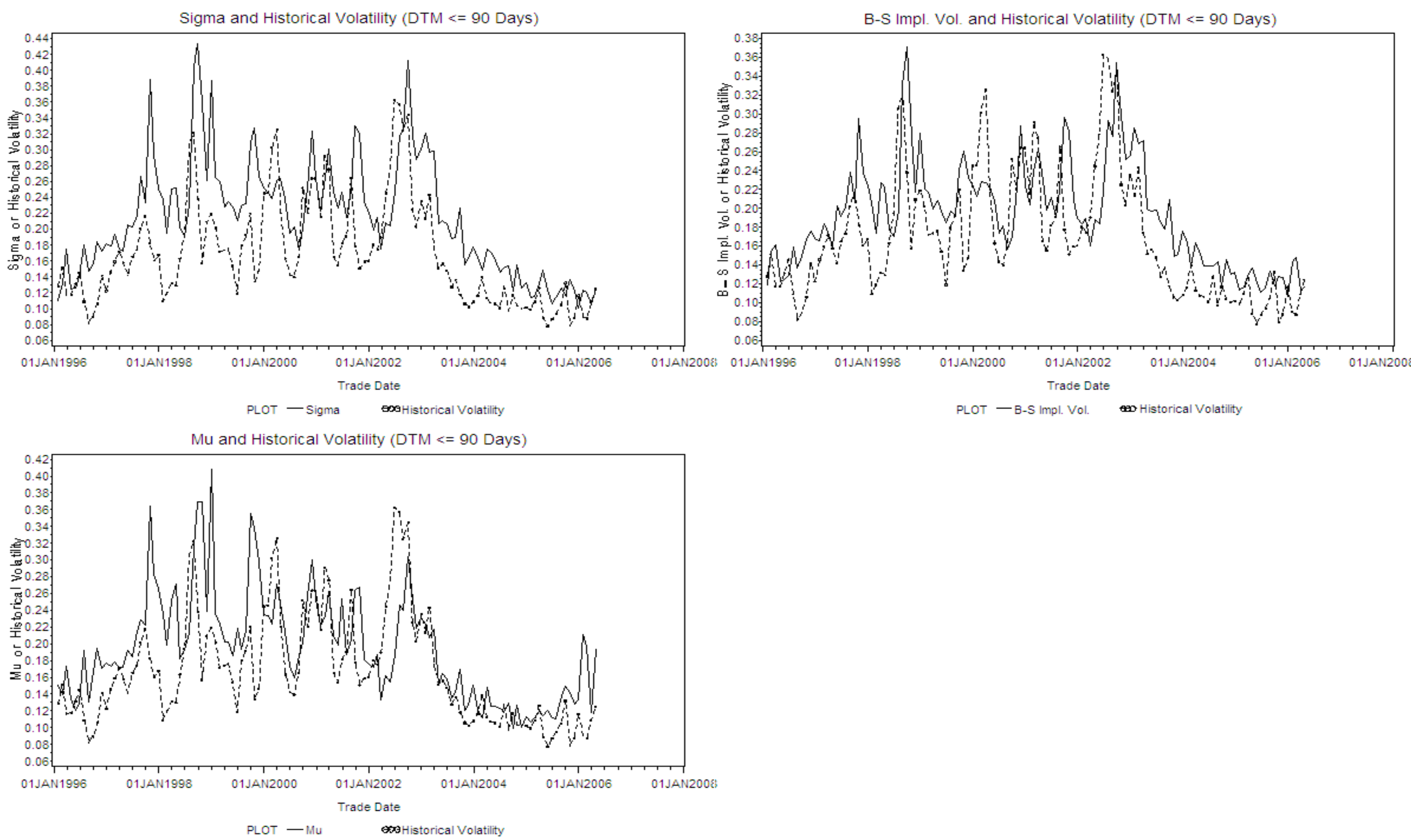
Panel A of Table 4 shows the regression results using all moneyness of S&P500 index call options.[30] As shown in this panel, the coefficients of $\overline{\sigma}_{t,T}^{BS}$, $\sigma_{t,T}$ and $\sigma_{t,T}^2$ are significant using models (R1) and (R2) respectively. However, the adjusted R-square is higher for the equation containing $\sigma_{t,T}$ and $\sigma_{t,T}^2$ for every maturity group. This shows, when we take all options, $\sigma_{t,T}$ provides a better forecast of historical volatility of the stock than the $\overline{\sigma}_{t,T}^{BS}$. To investigate the performance of $\sigma_{t,T}$ further we have similar regressions in Panel B and Panel C of Table 4. As we see in Panel B, for stock price/strike price between 0.95 and 1.05, the adjusted R-squares are not higher for the equations containing $\sigma_{t,T}$ and $\sigma_{t,T}^2$. However, the adjusted R-squares are higher for the equations containing $\sigma_{t,T}$ and $\sigma_{t,T}^2$ using far-the-money options.[31]

29. Nonetheless, to differentiate our measure of ex-post volatility from the Andersen et al. (2001) measure, we use the term "historical volatility" for our measure. The way we compute this historical volatility is explained in section III.C.1.

30. In all our samples, we do not include options that have zero trading volume.

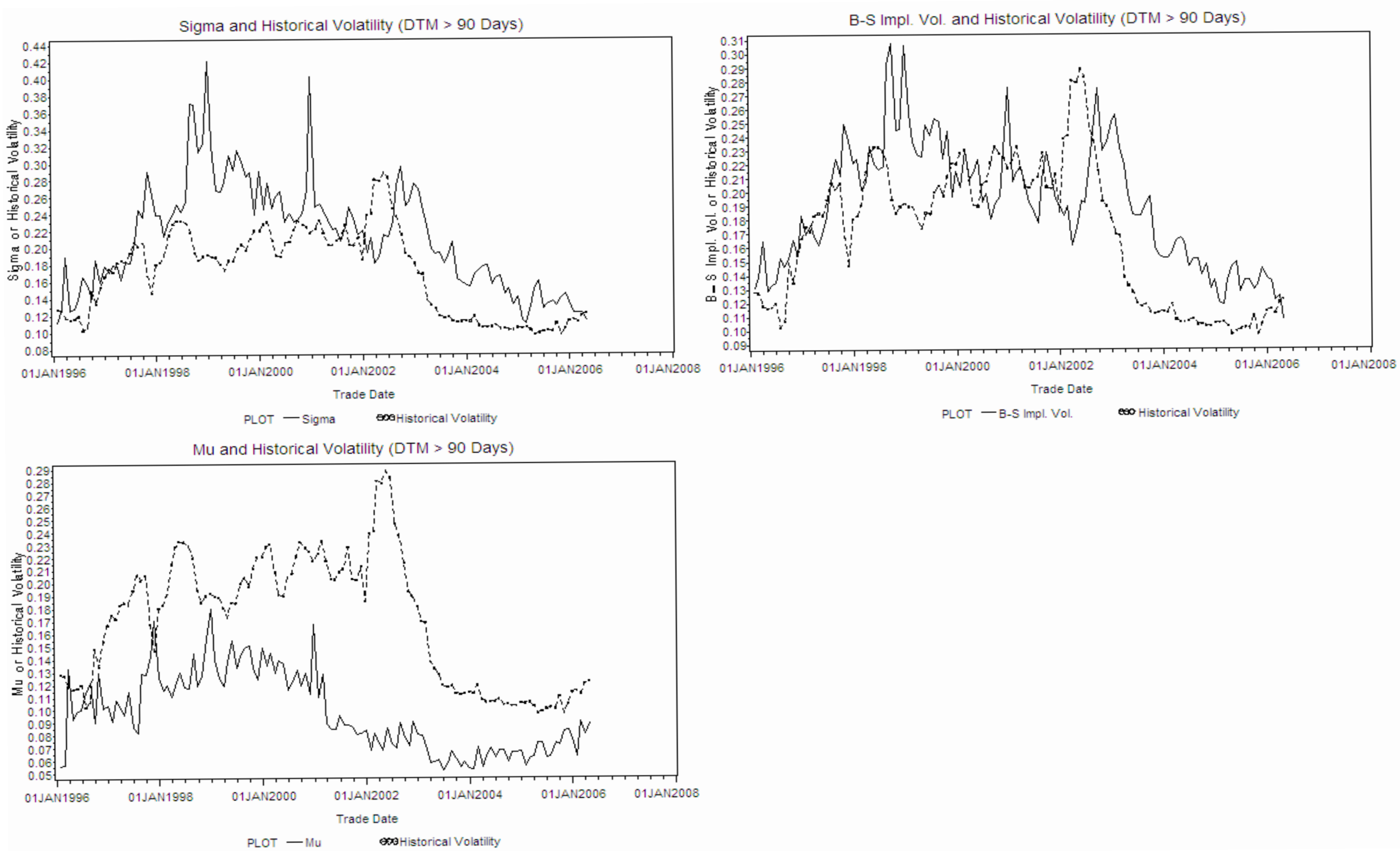
31. Options are defined to be far-the-money if the stock price divided by strike price is either higher than 1.05 or lower than 0.95.

Figure 2. 90 Days or less Predictability of Historical Volatility by Sigma, Mu, and B-S Implied Volatility of S&P500 Index.



Mu is option implied expected return, and Sigma is option implied volatility. Mu and Sigma are jointly estimated using risk adjusted model on options data.

Figure 3. More than 90 Days Predictability of Historical Volatility by Sigma, Mu, and B-S Implied Volatility of S&P500 Index.



Mu is option implied expected return, and Sima is option implied volatility. Mu and Sigma are jointly estimated using risk adjusted model on all moneyness call options data. DTM is days-to-maturity.

Table 4. Information Content of the Nested Model Using Mu, Sigma, and Black-Scholes Implied Volatility.

Panel A. SPX Call Options Using All Moneyness

Intercept	$\sigma_{t,T}$	$\sigma^2_{t,T}$	$\mu_{t,T}$	$\mu^2_{t,T}$	$\bar{\sigma}^{BS}_{t,T}$	$\bar{\sigma}^{BS^2}_{t,T}$	Adj. RP^{2P}	LR
Days-to Maturity of Less than or Equal to 90 Days								
-0.0299(-1.17)	1.2336** (4.82)	-1.2986* (-2.28)					0.4379	20.3474**
-0.0236(-0.79)					1.1905** (3.68)	-0.9544 (-1.18)	0.4187	32.4164**
0.0197(0.68)	1.9075** (4.67)	-2.9228** (-3.77)	0.0526 (0.36)	-0.2457 (-0.95)	-1.3344* (-2.49)	3.6476** (3.23)	0.4629	
Days-to Maturity of Greater than 90 Days								
-0.0681**(-4.45)	1.7558** (12.17)	-2.7056** (-8.89)					0.4434	13.9759**
-0.1891**(-6.84)					3.0648** (10.11)	-5.8532** (-7.67)	0.4328	22.1500**
-0.1912**(-4.42)	0.3136 (0.81)	-0.2417 (-0.35)	0.627* (2.08)	-2.4604 (-1.65)	2.4852** (3.5)	-5.2144** (-3.19)	0.456	
All Maturities								
-0.0461**(-2.89)	1.468** (9.25)	-1.9368** (-5.37)					0.4282	20.6095**
-0.0663**(-2.6)					1.6909** (5.96)	-2.2575** (-3.05)	0.4048	52.3657**
-0.0073(-0.28)	1.8209** (6.83)	-2.9905** (-6.38)	-0.0092 (-0.11)	-0.0526 (-0.3)	-0.8351 (-1.85)	2.5825* (2.49)	0.4401	

Table 4, continued. Information Content of the Nested Model Using Mu, Sigma, and Black-Scholes Implied Volatility.

Panel B. SPX Call Options Using StockPrice/StrikePrice between 0.95 and 1.05 (near the money)

Intercept	$\sigma_{t,T}$	$\sigma^2_{t,T}$	$\mu_{t,T}$	$\mu^2_{t,T}$	$\bar{\sigma}^{BS}_{t,T}$	$\bar{\sigma}^{BS^2}_{t,T}$	Adj. RP2P	LR
Days-to Maturity of Less than or Equal to 90 Days								
0(0)	0.9077** (4.55)	-0.6131 (-1.48)					0.4812	21.5486**
-0.0114(-0.43)					1.0455** (3.64)	-0.5166 (-0.73)	0.4853	18.6622**
0.0547(1.89)	2.1375** (3.76)	-3.8914** (-3.46)	-0.1606 (-1.27)	0.2987 (1.62)	-1.8916* (-2.14)	5.8757** (2.77)	0.5055	
Days-to Maturity of Greater than 90 Days								
0.0041(0.38)	0.9171** (12.63)	-0.7982** (-7.27)					0.4759	36.3098**
-0.193**(-5.73)					2.9321** (8.03)	-5.2307** (-5.88)	0.5191	12.4349*
-0.2158**(-5.78)	-0.1912 (-0.69)	0.1589 (0.72)	0.7401* (2.11)	-1.9228 (-1.61)	2.918** (6.76)	-5.1362** (-6.07)	0.5333	
All Maturities								
-0.0053(-0.63)	0.9771** (16.1)	-0.829** (-9.58)					0.4765	16.1417**
-0.0501(-1.94)					1.447** (5.04)	-1.5308* (-2.08)	0.4785	13.6985**
-0.0389 (-1.49)	0.53* (2.55)	-0.4676** (-2.62)	-0.0485 (-0.57)	0.2009 (1.42)	0.8354* (2.15)	-1.1626 (-1.45)	0.4863	

Table 4, continued. Information Content of the Nested Model Using Mu, Sigma, and Black-Scholes Implied Volatility.

Panel C. SPX Call Options Using StockPrice/StrikePrice of Less than 0.95 or Greater than 1.05 (far from the money)

Intercept	$\sigma_{t,T}$	$\sigma^2_{t,T}$	$\mu_{t,T}$	$\mu^2_{t,T}$	$\bar{\sigma}^{BS}_{t,T}$	$\bar{\sigma}^{BS^2}_{t,T}$	Adj. R^2	LR
Days-to Maturity of Less than or Equal to 90 Days								
-0.001(-0.08)	1.0099** (10.64)	-0.9703** (-8.19)					0.3331	11.6945*
-0.0146(-0.52)					1.2185** (4.17)	-1.3647 (-1.96)	0.2742	39.3134**
0.0246(0.91)	1.2148** (6.72)	-1.1504** (-6.76)	-0.0812 (-0.75)	-0.1393 (-1.12)	-0.4097 (-1.16)	0.9081 (1.23)	0.3485	
Days-to Maturity of Greater than 90 Days								
-0.0172(-1.12)	1.3344** (10.01)	-1.8246** (-6.63)					0.3912	9.8708*
-0.1313**(-5.71)					2.6113** (10.18)	-4.9591** (-7.61)	0.3808	15.2859**
-0.1274**(-3.7)	0.3576 (0.9)	-0.1559 (-0.21)	0.4193 (1.58)	-1.4697 (-1.16)	2.0244** (2.95)	-4.5422** (-2.86)	0.4021	
All Maturities								
0.0063(0.5)	1.0288** (10.17)	-1.0906**(-5.84)					0.3383	12.8482*
-0.0423(-1.96)					1.5663**(6.68)	-2.2206** (-3.69)	0.2972	51.9050**
0.0037(0.17)	1.0571** (6.79)	-1.0746**(-5.44)	-0.0591(-0.76)	-0.1118(-0.97)	0.0204(0.06)	0.0497(0.07)	0.3473	

This table presents the generalized method of moments regression for forecast of historical volatility using $\mu_{t,T}$, $\sigma_{t,T}$, and Black-Scholes implied volatility ($\bar{\sigma}^{BS}_{t,T}$) for different maturity groups for the period of January 1996-April 2006. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group. Values in parenthesis are *t*-statistics. Dependent variable is historical volatility of the index for the period of the option. $\bar{\sigma}^{BS}_{t,T}$ is Black-Scholes implied volatility, $\mu_{t,T}$ and $\sigma_{t,T}$ are the estimated values from our model. For S&P 500 index (SPX), stock price is the level of the index. LR is the likelihood ratio to test whether the restricted regressions are valid. ** and * represent the *p*-values of less than 0.01, and between 0.01 and 0.05, respectively.

This shows $\sigma_{t,T}$ provides a better representation of ex-ante volatility than $\bar{\sigma}_{t,T}^{BS}$ using the information in far-from-the-money options. Even though $\bar{\sigma}_{t,T}^{BS}$ does better when we take only near-the-money options, it is unable to provide a single implied volatility measure that we can use for options of all moneyness. On the other hand, $\sigma_{t,T}$ provides a better measure of ex-ante volatility that can be used for options of all moneyness.

How does equation (R1) compare with equation (R3) in explaining the historical volatility? To address this question first we see for all panels using near-the-money, all moneyness, and far-the-money options, the adjusted R-square is higher for the unrestricted regression (R3) as shown in Table 4. For example, in Panel A for <=90 days-to-maturity group, the adjusted R-square for the unrestricted model (R3) is 46.29%, and for the restricted model (R1) it is 41.87%. This shows that equation (R3) provides a better model, such that it has a higher adjusted R-square for near-the-money, far-the-money, and options of all moneyness. Second, for all maturities the coefficients of $\sigma_{t,T}$ and $\sigma_{t,T}^2$ are significant for all panels of Table 4 in the unrestricted equation (R3). However that is not the case with $\bar{\sigma}_{t,T}^{BS}$. For example, in Panel A and Panel C the coefficients of $\bar{\sigma}_{t,T}^{BS}$ are not significant.

Finally, we use the likelihood ratio to test the hypothesis H_0: $a_{43} = a_{44} = a_{45} = a_{46} = 0$. The likelihood ratios are significant in our experiment for all panels of Table 4. Therefore, we reject the restricted model as given in equation (R1) for all maturity groups shown in this table. This result indicates that the inclusion of $\mu_{t,T}$ and $\sigma_{t,T}$ and their second-order terms provides a better model than simply using Black-Scholes implied volatility to forecast the historical volatility for all near and far maturity groups, and for any moneyness level.

2. Information Content of Non-Nested Models

In this subsection we compare the non-nested models that have only the risk-adjusted variables ($\mu_{t,T}$, $\sigma_{t,T}$, and the square terms) or the $\bar{\sigma}_{t,T}^{BS}$ variable (and its square term) to forecast historical volatility. We use two different variations of *J*-test that are popularly used in the literature.

The non-nested models that we use to forecast historical volatility can be given by the following regressions:

$$\sigma_{t,T}^{HV} = \alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS^2} + \omega_{1t,T} \quad \text{(R4)}$$

$$\sigma_{t,T}^{HV} = \alpha_{20} + \alpha_{21}\sigma_{t,T} + \alpha_{22}\sigma_{t,T}^2 + \omega_{2t,T} \quad \text{(R5)}$$

$$\sigma_{t,T}^{HV} = \alpha_{30} + \alpha_{31}\sigma_{t,T} + \alpha_{32}\sigma_{t,T}^2 + \alpha_{33}\mu_{t,T} + \alpha_{34}\mu_{t,T}^2 + \omega_{3t,T} \quad \text{(R6)}$$

To compare (R5) or (R6) with (R4) we take the fitted values of $\sigma_{t,T}^{HV}$ from these equations and use the following *J*-test regressions:

$$\sigma_{t,T}^{HV} = \phi_1[\alpha_{20} + \alpha_{21}\sigma_{t,T} + \alpha_{22}\sigma_{t,T}^2] + (1-\phi_1)[\sigma_{t,T}^{HV} - \omega_{1t,T}] + e_{t,T} \quad \text{(R7)}$$

$$\sigma_{t,T}^{HV} = \phi_1[\alpha_{30} + \alpha_{31}\sigma_{t,T} + \alpha_{32}\sigma_{t,T}^2 + \alpha_{33}\mu_{t,T} + \alpha_{34}\mu_{t,T}^2] + (1-\phi_1)[\sigma_{t,T}^{HV} - \omega_{1t,T}] + e_{t,T} \quad \text{(R8)}$$

$$\sigma_{t,T}^{HV} = \phi_2[\alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS^2}] + (1-\phi_2)[\sigma_{t,T}^{HV} - \omega_{2t,T}] + e_{t,T} \quad \text{(R9)}$$

Using (R7) and (R9), we can test whether the Black-Scholes implied standard deviation offers any incremental information over risk-adjusted implied volatility. If the Black-Scholes model does not have any incremental information, then ϕ_1 should be close to 1 and significant, and ϕ_2 should be insignificant.[32] To find whether ϕ_1 is in fact 1, we test the null hypothesis of H_0: $\phi_1 = 1$. Since our null hypothesis is the result intended, in this test, to minimize the Type II error the critical *p*-value should be higher.[33] The left side of Table 5 shows the results of this comparison. As we see from left side of Panel A using all moneyness, ϕ_2 is insignificant for <=90 days-to-maturity group. Also, ϕ_1 is significant and we fail to reject the null hypothesis that $\phi_1 = 1$ for this maturity group. This shows that for <=90 days-to-maturity group Black-Scholes implied standard deviation provides no incremental information over our implied volatility. However, for >90 days-to-maturity group we cannot say that the Black-Scholes implied standard deviation provides no incremental information over the risk-adjusted $\sigma_{t,T}$. Results are similar when we take both $\mu_{t,T}$ and $\sigma_{t,T}$ to compare with the Black-Scholes implied standard deviation. Even for the near-the-money options (Panel B) for <=90 days-to-maturity group, ϕ_2 is insignificant, and we fail to reject the null hypothesis that $\phi_1 = 1$. This indicates that even when we do not have a volatility smile the risk-adjusted $\sigma_{t,T}$ performs marginally better than $\bar{\sigma}_{t,T}^{BS}$. Furthermore, as we see from Panel C of Table 5, consistent with the prior literature, when we have many far-from-the-money options, $\bar{\sigma}_{t,T}^{BS}$ does not provide any incremental information. These results suggest, to forecast volatility for shorter maturity of 90-days or less, the risk-adjusted $\sigma_{t,T}$ provides a better alternative over the $\bar{\sigma}_{t,T}^{BS}$ for any moneyness level. Furthermore, if we have many far-from-the-money options, then $\sigma_{t,T}$ is a better choice irrespective of days-to-maturity.

We also test another variation[34] of the above *J*-test using the following regressions:

$$\sigma_{t,T}^{HV} = (1-\psi_2)[\alpha_{20} + \alpha_{21}\sigma_{t,T} + \alpha_{22}\sigma_{t,T}^2] + \psi_2[\sigma_{t,T}^{HV} - \omega_{1t,T}] + e_{t,T} \quad \text{(R10)}$$

32. Our discussions compare (R5) with (R4). However, we can also compare (R6) with (R4) to find if Black-Scholes implied standard deviation offers any incremental information over risk-adjusted $\sigma_{t,T}$ and $\mu_{t,T}$. In that case we use (R8) instead of (R7) and (R9) is given by:

$$\sigma_{t,T}^{HV} = \phi_2[\alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS^2}] + (1-\phi_2)[\sigma_{t,T}^{HV} - \omega_{3t,T}] + e_{t,T} \quad \text{(R9)}$$

We show the results for both (R5), (R4) comparison, and (R6), (R4) comparison in Table 6.

33. We take 5% significance level as the cutoff point, approximately in the middle of 10% and 1%.

34. Davidson and MacKinnon (1981).

$$\sigma_{t,T}^{HV} = (1-\psi_2)[\alpha_{30} + \alpha_{31}\sigma_{t,T} + \alpha_{32}\sigma_{t,T}^2 + \alpha_{33}\mu_{t,T} + \alpha_{34}\mu_{t,T}^2] + \psi_2[\sigma_{t,T}^{HV} - \omega_{1t,T}] + e_{t,T} \quad \text{(R11)}$$

$$\sigma_{t,T}^{HV} = (1-\psi_1)[\alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS^2}] + \psi_1[\sigma_{t,T}^{HV} - \omega_{2t,T}] + e_{t,T} \quad \text{(R12)}$$

For Black-Scholes implied volatility not to have any incremental contribution to forecast historical volatility, ψ_2 should be insignificant in (R10) and ψ_1 should be significant and closer to 1 in (R12).[35] Similar to the prior *J*-test, we test the null hypothesis that H_0: $\psi_1 = 1$. The results are given on the right side of Table 5. The results using this alternative *J*-test are mostly similar to the prior *J*-test. Consistent with the prior *J*-test, when we take any moneyness for near term options (90-days or less), our results show Black-Scholes implied standard deviation does not contain incremental information beyond the risk-adjusted $\sigma_{t,T}$ (or $\mu_{t,T}$ and $\sigma_{t,T}$). However, for far term options (more than 90-days), we cannot argue that $\sigma_{t,T}$ (or $\mu_{t,T}$ and $\sigma_{t,T}$) alone is sufficient to forecast historical volatility. Nonetheless, in this case we can still use the unrestricted regression using all the three variables which provide a better model for all near and far maturity groups, and for any moneyness level as we find in Table 4.

In general, our risk-adjusted approach provides a better measure (than $\bar{\sigma}_{t,T}^{BS}$) that captures moneyness biases even without adjusting for stochastic volatility. Our results are stronger in forecasting the short term volatility for 90-days or less. Therefore, if we are concerned about the smile while forecasting historical volatility using all options data, then our approach provides a better solution than $\bar{\sigma}_{t,T}^{BS}$ so that we do not need any adjustment for moneyness bias.

D. Measurement Error and Robustness Checks

Option spread and option volume could be one possible reason for the term structure of $\mu_{t,T}$.[36] As we see in Table 1, spread and option volume are lower for higher days-to-maturity.[37] This experiment is also motivated by the findings of Longstaff (1995). Using S&P100 index options and Black-Scholes (1973) risk-neutral valuation, Longstaff shows that the implied cost of the index is significantly higher in the option market than in the stock market. The author also shows the percentage pricing difference between the implied and actual index is directly related to the measures of transaction costs and liquidity such as the option spread, volume,

35. If we use risk-adjusted $\sigma_{t,T}$ and $\mu_{t,T}$ instead of just $\sigma_{t,T}$, then we use (R11) instead of (R10) and (R12) will be given by:

$$\sigma_{t,T}^{HV} = (1-\psi_1)[\alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS2}] + \psi_1[\sigma_{t,T}^{HV} - \omega_{3t,T}] + e_{t,T} \quad \text{(R12)}$$

36. Term structure of $\mu_{t,T}$ is the value of $\mu_{t,T}$ for different option maturity date of *T*, for a given option pricing date of *t*.

37. When we take finer groups, such as 30, 60, 90 days-to-maturity groups, we clearly see the average volume and spread decrease with days-to-maturity.

Table 5. Comparison of Non-Nested Models Using Mu, Sigma, or Black-Scholes Implied Volatility.

SPX Calls	Days to maturity	ϕ_1 Coeff.(*p*-val)	$H_0: \phi_1=1$ χ^2 LR (*p*-val)	ϕ_2 Coeff.(*p*-val)	ψ_1 Coeff.(*p*-val)	$H_0: \psi_1=1$ χ^2 LR (*p*-val)	ψ_2 Coeff.(*p*-val)
Panel A. using all moneyness							
Sigma, and square term	<= 90 days	0.6797(0.004)	1.87(0.172)	0.1218(0.558)	0.8781(0.000)	2.82(0.093)	0.3202(0.172)
	> 90 days	0.6044(0.002)	4.03(0.044)	0.3594(0.067)	0.6405(0.001)	4.60(0.032)	0.3955(0.045)
	All Maturities	0.7496(0.000)	2.48(0.115)	0.1313(0.366)	0.8686(0.000)	2.43(0.118)	0.2503(0.115)
Sigma, Mu, and square terms	<= 90 days	0.5459(0.036)	3.03(0.081)	0.1429(0.390)	0.8570(0.000)	3.00(0.083)	0.4540(0.082)
	> 90 days	0.3288(0.115)	10.37(0.001)	0.3639(0.043)	0.6360(0.000)	6.00(0.014)	0.6711(0.001)
	All Maturities	0.7364(0.000)	2.64(0.104)	0.1274(0.367)	0.8725(0.000)	2.47(0.116)	0.2635(0.104)
Panel B. using moneyness between 0.95 and 1.05							
Sigma, and square term	<= 90 days	0.3576(0.343)	2.90(0.088)	0.5563(0.111)	0.4436(0.203)	2.99(0.084)	0.6423(0.089)
	> 90 days	0.2142(0.194)	22.74(0.000)	0.5528(0.008)	0.4471(0.031)	31.74(0.000)	0.7857(0.000)
	All Maturities	0.4543(0.024)	7.39(0.006)	0.5263(0.006)	0.4736(0.014)	7.88(0.005)	0.5456(0.006)
Sigma, Mu, and square terms	<= 90 days	0.2901(0.500)	2.72(0.099)	0.4320(0.118)	0.5679(0.040)	2.58(0.108)	0.7098(0.100)
	> 90 days	0.0130(0.931)	41.99(0.000)	0.5335(0.006)	0.4664(0.017)	31.16(0.000)	0.9869(0.000)
	All Maturities	0.5318(0.010)	5.09(0.024)	0.3941(0.044)	0.6058(0.002)	5.82(0.016)	0.4681(0.024)

Table 5, continued. Comparison of Non-Nested Models Using Mu, Sigma, or Black-Scholes Implied Volatility.

		ϕ_1	$H_0: \phi_1=1$	ϕ_2	ψ_1	$H_0: \psi_1=1$	ψ_2
SPX Calls	Days to maturity	Coeff.(*p*-val)	LR χ^2(*p*-val)	Coeff.(*p*-val)	Coeff.(*p*-val)	LR χ^2(*p*-val)	Coeff.(*p*-val)
Panel C. using moneyness not between 0.95 and 1.05							
Sigma, and square term	<= 90 days	0.9156(0.000)	0.14(0.703)	0.0478(0.808)	0.9521(0.000)	0.54(0.462)	0.0843(0.704)
	> 90 days	0.5937(0.007)	3.42(0.064)	0.3278(0.184)	0.6721(0.006)	5.67(0.017)	0.4063(0.065)
	All Maturities	0.8518(0.000)	0.80(0.372)	0.1193(0.463)	0.8806(0.000)	1.06(0.303)	0.1481(0.372)
Sigma, Mu, and square terms	<= 90 days	0.9845(0.000)	0.01(0.940)	-0.0360(0.815)	1.0360(0.000)	1.79(0.180)	0.0154(0.940)
	> 90 days	0.4363(0.060)	5.91(0.015)	0.3226(0.160)	0.6773(0.003)	4.26(0.039)	0.5636(0.015)
	All Maturities	0.9444(0.000)	0.14(0.711)	0.0401(0.754)	0.9598(0.000)	0.14(0.709)	0.0555(0.711)

This table presents the generalized method of moments regression to compare non-nested model of Black-Scholes volatility (and the square term) with the $\sigma_{t,T}$ (and the square term) or $\mu_{t,T}$ and $\sigma_{t,T}$ (and the square terms) using month end data for different maturity groups for the period of January 1996-April 2006. The left and right side panel provide two different versions of the *J*-test as described in section 4.3.2. In the first version (left side panel) of the *J*-test regression, $(1 - \phi_1)$ is the coefficient of the fitted value from the Black-Scholes implied standard deviation (and the square term) non-nested equation; $(1 - \phi_2)$ is the coefficient of the fitted value from $\sigma_{t,T}$ and the square term (or $\mu_{t,T}$ and $\sigma_{t,T}$ and the square terms) non-nested equation. In the second version (right side panel) of the *J*-test regression, ψ_1 is the coefficient of the fitted value from $\sigma_{t,T}$ and the square term (or $\mu_{t,T}$ and $\sigma_{t,T}$ and the square terms) non-nested equation; ψ_2 is the coefficient of the fitted value from Black-Scholes implied standard deviation and the square term non-nested equation. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in <=90 days-to-maturity group. If days to maturity is greater than 90 days it is in > 90 days-to-maturity group. Values in parenthesis are *p*-values. Dependent variable is historical volatility of the index for the period of the option. χ^2 is based on the likelihood ratio tests.

Table 6. Results from Regressing Level of Mu on the Indicated Variables.

Days-to-maturity groups	<= 90 Days	> 90 Days	All Maturities
Intercept	-0.3439(-1.95)	-0.0548*(-2.31)	-0.1674**(-5.56)
AvgMoneyness	0.6083**(3.42)	0.1732**(7.49)	0.2641**(8.64)
DaysToMaturity	-0.002**(-8.16)	-1.10E-04**(-9.55)	-1.10E-04**(-9.11)
AbsRet	2.6419**(4.64)	1.2907**(5.76)	1.3217**(3.69)
LAbsRet	2.3448**(4.25)	1.1145**(4.64)	1.1902**(3.58)
L2AbsRet	1.3331**(2.65)	-0.3074(-1.57)	0.122(0.37)
AvgSpread	-0.0264(-0.65)	0.0989*(2.54)	0.1085**(2.73)
TotalOpnInt	-1.14E-07**(-3.91)	-1.04E-07**(-2.71)	-2.10E-07**(-6.34)
AvgVolume	-1.00E-05(-1.74)	1.25E-06(0.47)	3.00E-06(1.02)
RecCount	0.0011(0.94)	1.85E-04(0.25)	0.0061**(7.25)
Ret	-0.1874(-0.46)	-0.0695(-0.4)	0.0063(0.03)
LRet	-0.219(-0.62)	-0.0729(-0.48)	-0.0236(-0.11)
L2Ret	-0.0687(-0.2)	0.0872(0.63)	-0.0751(-0.35)
Adj-R2	0.5142	0.4554	0.5843

This table presents results from regression of $\mu_{t,T}$ levels for different days-to-maturity groups using S&P500 index all moneyness call option data. We use generalized method of moments for this estimation. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in <=90 days-to-maturity group. If days to maturity is greater than 90 days it is in > 90 days-to-maturity group. The values in parenthesis are the *t*-statistics. AvgMoneyness is average of the stock price divided by the strike price of options used to compute $\mu_{t,T}$. For S&P500, stock price is the level of the index. AbsRet, LAbsRet, L2AbsRet are the current and first two lagged daily absolute returns of the S&P 500 index. AvgSpread is average of (offer-bid)/call price of all option records used to compute $\mu_{t,T}$. TotalOpnInt is the total option interest of the options used to compute $\mu_{t,T}$. AvgVolume is the average volume, and RecCount is the number of records used to compute $\mu_{t,T}$. Ret, LRet, L2Ret are the current and first two lagged daily returns of the S&P 500 index. ** and * represent the *p*-values of less than 0.01, and between 0.01 and 0.05, respectively.

and open interest. To examine the possible influence of these market friction proxies on the term structure of $\mu_{t,T}$, we regress $\mu_{t,T}$ on transaction cost proxy that is given by the average spread, and liquidity measures that are given by average volume and total open interest. We also control for other findings of pricing biases of Black-Scholes model. These findings include Chiras and Manaster (1978), Macbeth and Merville (1980), Rubenstein (1985), and Canina and Figlewski (1993). These studies find three types of pricing bias in Black-Scholes model, namely a time to expiration bias, a moneyness bias, and a volatility bias. To control for these biases we include the time to expiration, moneyness (stock price/strike price), and current and first two lagged values of absolute daily returns. To control for volatility bias, we use current and first two lagged values of absolute daily returns instead of implied volatility $\sigma_{t,T}$ since this parameter is jointly estimated with $\mu_{t,T}$, which can induce spurious correlation.

Further, we use number of calls to compute $\mu_{t,T}$ and $\sigma_{t,T}$ as a measure of trading activity, current and lagged daily returns as a measure of path-dependent effects (Leland (1985). The results are shown in Table 6. The regression results provide mixed evidence that term structure of $\mu_{t,T}$ is related to the market friction proxies namely spread, volume, and open interest. For example, for >90 days-to-maturity group the coefficient of average spread and total open interest are 0.0989 and -1.04E-07, respectively, and are significant, whereas average volume is not significant. Similarly, for <=90 days-to-maturity group only total open interest is significant. Interestingly, coefficient of total open interest is negative and significant for all maturity groups. However, in the data, total open interest does not increase (as the days to maturity increases) to support the declining term structure of $\mu_{t,T}$.[38] As we see average spread is not significant for <=90 days to maturity groups, that means spread cannot explain the sharp term structure of $\mu_{t,T}$, especially for the lower days-to-maturity group as seen in Figure 1. Therefore, our evidence shows that friction proxies are not the cause of the term structure of $\mu_{t,T}$.[39]

Our modified risk-adjusted approach can be questionable in a framework with stochastic volatility and jumps, which means we may not be using the exact model of option pricing. Many of the past literature, for example, Merton (1976), Cox and Ross (1976), Hull and White (1987), Scott (1987), and Heston (1993a), extend basic Black-Scholes (B-S) model to incorporate jumps and stochastic volatility. However, the risk-adjusted formulas we use do not have these adjustments and assumes a lognormal diffusion process. This can create errors-in-variable problem in implied return and implied volatility computation. To minimize the effect of errors-in-variable bias, we alternatively take options, which are only near-the-money (stock price divided by strike price is between 0.95 and 1.05).[40] We still see a strong term structure of $\mu_{t,T}$ in this case. Moreover, we do not take options that do not have any trading in a given day. We also separately estimate $\mu_{t,T}$ and $\sigma_{t,T}$ for deep-in-the-money call options where stock price divided by strike price is greater than 1.20, and deep-out-of-the-money call options where stock price divided by strike price is less than 0.90. In both cases, we still get the term structure of $\mu_{t,T}$ (not shown here). Measurement error may be systematically affected by time-to-maturity (Canina and Figlewski 1993). To mitigate these errors, options with same days-to-maturity are used to compute implied expected return and implied volatility. It may also be possible to have systematic bias in our computation due to other factors such as the market friction (Longstaff 1995) proxies. To examine this possibility, we regress $\mu_{t,T}$ on these proxies to show in the previous paragraph that they do not explain the term structure of $\mu_{t,T}$.

38. Open interest is mostly lower for higher days to maturity.

39. Table 4 is based on all moneyness of S&P500 index options. When we take only near-the-money (stock price divided by strike price is between 0.95 and 1.05), the evidence of friction proxies on $\mu_{t,T}$ are much weaker; however, we still see a very strong term structure of $\mu_{t,T}$ even in this case.

40. The term structure of $\mu_{t,T}$ using near-the-money is not shown here.

Furthermore, our procedure might have problems of computing European option prices from OptionMetrics implied volatility and using that to compute our implied return and implied volatility. As a part of our robustness check, we show that, even if we use different methods to compute option prices, the term structure of implied expected return remains in our result. For example, in our main result we compute the European price using the OptionMetrics implied volatility adjusted for dividends. If this price is higher than the bid-ask midpoint, then we take the bid-ask midpoint, or else we take the European price as the option price for $\mu_{t,T}$ and $\sigma_{t,T}$ estimation. In our robustness check, we compute $\mu_{t,T}$ and $\sigma_{t,T}$ first by taking the European price, and then by taking the bid-ask midpoint price as the option price and we get clear term structures of implied expected return in both cases.

IV. CONCLUSION

This paper uses a risk-adjusted method for joint estimation of implied expected stock return and volatility from market observed option prices. We find that investors in option markets have a higher expectation of stock return in the short-term, but a lower expectation of stock return in the long-term. This term structure of expected stock return also remains for deep-in and deep-out of the money call options (not shown here). We also find that the market friction proxies such as volume, open interest, and bid-ask spread do not explain this term structure. It also persists for combined call and put options (not shown here). This term structure finding supports McNulty et al.'s (2002) explanation, where the authors argue that shorter horizon investments should be discounted at a higher rate. However, they use a heuristic approach without a theoretical setting to arrive at these results.

On the other hand, our paper provides the necessary theoretical support for this finding. Using all moneyness options, we further find that the term structure of our volatility is "flatter" than the term structure of Black-Scholes implied standard deviation. We also find that the implied volatility ($\sigma_{t,T}$) provides a better model than Black-Scholes implied standard deviation ($\overline{\sigma}_{t,T}^{BS}$) to forecast historical volatility for maturities of 90-days or less for any moneyness level. In general, our risk-adjusted approach provides a better measure (than $\overline{\sigma}_{t,T}^{BS}$) that captures moneyness biases even without adjusting for stochastic volatility. Therefore, if we are concerned about the smile while forecasting historical volatility using all options data, then our approach provides a better solution than $\overline{\sigma}_{t,T}^{BS}$ so we do not need any adjustment for moneyness bias. In addition, we find that a combination of our implied expected return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) with $\overline{\sigma}_{t,T}^{BS}$ provides a better model than using $\overline{\sigma}_{t,T}^{BS}$ alone to forecast future volatility for all near and far maturity groups, and for any moneyness level.

These findings may provide a starting point for further research. For example, our approach may be used to estimate the cost of equity for different industry portfolios. Especially, estimates of expected return for one-year or more will have lower standard error, which is a necessary condition for this to be useful as an estimate of cost of equity. Using this approach, we can compute the expected return of any individual stock without using any information of the market portfolio such as the market risk premium. Moreover, our results can be deduced without assuming a utility structure for the representative agent. Furthering the research, we plan to investigate whether the term structure persists using other approaches. Nonetheless, better forecasting capability of future volatility using our sigma and expected return might suggest additional investigation of information content in these findings.

APPENDIX

A.1 Risk-Adjusted Formulas for Call Options

A.1.1 Proof of Proposition 1

We prove the proposition without assuming the CAPM.

Let the price change for the stock and option during a small interval of time Δt are ΔS and ΔC, respectively. Without loss of generality, we assume t as the current time. Let the current stock and option prices are S_t and C_t , respectively. This implies:

$$\begin{aligned} \frac{\Delta S}{S_t} &= r_S \\ \frac{\Delta C}{C_t} &= r_C \\ E[r_S] &= \mu \Delta t \\ E[r_C] &= k \Delta t \end{aligned} \tag{A1}$$

When Δt is a small interval of time, then Δt tends to dt, ΔS tends to dS, and ΔC tends to dC.

Since stock price S follows a geometric Brownian, the change in the price of the stock ΔS during the small interval of time Δt is:

$$dS = \mu S_t dt + \sigma S_t dW \tag{A2}$$

where dW is the Wiener differential. Then, following Ito's Lemma, option price change is given by:

$$\begin{aligned} dC &= \frac{\partial C}{\partial S} dS + \left(\frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S_t^2 + \frac{\partial C}{\partial t} \right) dt \\ &= \frac{\partial C}{\partial S} dS + \left(rC_t - \frac{\partial C}{\partial S} rS_t \right) dt \end{aligned} \tag{A3}$$

where the second line of (A3) is derived from the Black-Scholes PDE (partial differential equation). From (A3), we can then compute the covariance between the option return and the stock return as follows:

$$\begin{aligned}\text{cov}\left[\frac{dC}{C_t},\frac{dS}{S_t}\right] &= \frac{1}{C_tS_t}\text{cov}[dC,dS] \\ &= \frac{1}{C_tS_t}\frac{\partial C}{\partial S}\text{var}[dS] \\ &= \frac{S_t}{C_t}\frac{\partial C}{\partial S}\text{var}\left[\frac{dS}{S_t}\right]\end{aligned} \tag{A4}$$

Then it follows that:

$$\begin{aligned}\frac{S_t}{C_t}\frac{\partial C}{\partial S} &= \frac{\text{cov}\left[\frac{dC}{C_t},\frac{dS}{S_t}\right]}{\text{var}\left[\frac{dS}{S_t}\right]} \\ &= \beta\end{aligned} \tag{A5}$$

Finally, taking the expectation of (A3), we obtain:

$$kdt = \beta\mu dt + r(1-\beta)dt \tag{A6}$$

Q.E.D.

Further, we note that, if we take covariance of both sides of (A3) with respect to the market return r_M, then we will obtain the following:

$$k = r + \beta(\mu - r) \tag{A7}$$

where

$$\beta = \frac{\beta_C}{\beta_S}$$

$$\beta_C = \frac{\text{cov}(r_C, r_M)}{\text{var}(r_M)}$$

$$\beta_S = \frac{\text{cov}(r_S, r_M)}{\text{var}(r_M)}$$

This implies:

$$\begin{aligned}\beta &= \frac{\text{cov}(r_C, r_S)}{\text{var}(r_S)} \\ &= \frac{\text{cov}(r_C, r_M)}{\text{cov}(r_S, r_M)}\end{aligned} \tag{A8}$$

A.1.2. Proof of Proposition 2

For readability we drop the subscript *t,T* for μ, σ, and *k*; and subscript *t,T, K* for β during this proof. From (5a), we can compute the expected value of the call payoff using the risk-adjusted measure as:

$$\begin{aligned} E[C_T] &= e^{k(T-t)}C_t \\ &= S_t e^{\mu(T-t)} N(h_1) - KN(h_2) \end{aligned} \tag{A9}$$

From the known result of the moment generating function of a Gaussian variable, we have:

$$\begin{aligned} \text{var}[S_T] &= E[S_T^2] - (E[S_T])^2 \\ &= S_t^2 e^{(2\mu+\sigma^2)(T-t)} - S_t^2 e^{2\mu(T-t)} \\ &= S_t^2 e^{2\mu(T-t)}\left(e^{\sigma^2(T-t)} - 1\right) \end{aligned} \tag{A10}$$

and

$$\begin{aligned} E[S_T C_T] &= \int_0^\infty S_T \max\{S_T - K, 0\}\phi(S_T)dS_T \\ &= \int_K^\infty S_T^2 \phi(S_T)dS_T - K\int_K^\infty S_T\phi(S_T)dS_T \\ &= S_t^2 e^{(2\mu+\sigma^2)(T-t)} N(h_3) - KS_t e^{\mu(T-t)} N(h_1) \end{aligned} \tag{A11}$$

where

$$h_3 = \frac{\ln S - \ln K + \left(\mu + \frac{3}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Hence, the covariance term in (7a) can be computed as:

$$\begin{aligned} \text{cov}\left[S_T, C_T\right] &= E[S_T C_T] - E[S_T]E[C_T] \\ &= S_t^2 e^{(2\mu+\sigma^2)(T-t)} N(h_3) - KS_t e^{\mu(T-t)} N(h_1) - S_t e^{\mu(T-t)}\left[S_t e^{\mu(T-t)} N(h_1) - KN(h_2)\right] \\ &= S_t^2 e^{2\mu(T-t)}\left[e^{\sigma^2(T-t)} N(h_3) - \frac{K}{S_t} e^{-\mu(T-t)}\left(N(h_1) - N(h_2)\right) - N(h_1)\right] \end{aligned} \tag{A12}$$

Finally, combining equations (7a), (A10), and (A12) we have:

$$\beta = \frac{S_t\left[e^{\sigma^2(T-t)} N(h_3) - \frac{K}{S_t} e^{-\mu(T-t)}\left(N(h_1) - N(h_2)\right) - N(h_1)\right]}{C_t\left(e^{\sigma^2(T-t)} - 1\right)} \tag{A13}$$

With the subscripts *t,T* attached to the parameters, equation (A13) can be written as equation (8) of Proposition (2).
Q.E.D.

A.2 Risk-Adjusted Formulas for Put Options

A.2.1 Proposition 1 for put options

Assume stock price S follows a geometric Brownian motion with an annualized expected instantaneous return of μ and volatility of σ. Let a put option on the stock at any point in time *t* is given by *P*(*S*,*t*) that matures at time *T*. Let *k* be the annualized expected instantaneous return on this option. Then for a small interval of time Δt, the relationship between μ and *k* can be given by:

$$k = r + \beta(\mu - r) \tag{A14}$$

where

$$\beta = \frac{\mathrm{cov}(r_P, r_S)}{\mathrm{var}(r_S)} \tag{A15}$$

and $r_s = \Delta S / S$ and $r_P = \Delta P / P$ are two random variables representing the stock return and put option return respectively during the period Δt . And *r* is the annualized constant risk-free rate for the period of the option.

Proof: The proof is similar to proposition 1.

As in proposition 1, we have:

$$\begin{aligned} \frac{\Delta S}{S} &= r_S \\ \frac{\Delta P}{P} &= r_P \\ E[r_S] &= \mu \Delta t \\ E[r_P] &= k \Delta t \end{aligned} \tag{A16}$$

When Δt is a small interval of time, then Δt approaches *dt*, ΔS approaches *dS*, and ΔP approaches *dP*.

Since stock price *S* follows a geometric Brownian, the change in the price of the stock ΔS during the small interval of time Δt is:

$$dS = \mu S_t dt + \sigma S_t dW \tag{A17}$$

where *dW* is the Wiener differential. Then, following Ito's Lemma, option price change is given by:

$$\begin{aligned} dP &= \frac{\partial P}{\partial S} dS + \left(\frac{1}{2} \frac{\partial^2 P}{\partial S^2} \sigma^2 S_t^2 + \frac{\partial P}{\partial t} \right) dt \\ &= \frac{\partial P}{\partial S} dS + \left(r P_t - \frac{\partial P}{\partial S} r S_t \right) dt \end{aligned} \tag{A18}$$

where the second line of (A18) is derived from the Black-Scholes PDE (partial differential equation). From (A18), we can then compute the covariance between the option return and the stock return as follows:

$$\begin{aligned} \operatorname{cov}\left[\frac{dP}{P_t}, \frac{dS}{S_t}\right] &= \frac{1}{P_t S_t}\operatorname{cov}[dP, dS] \\ &= \frac{1}{P_t S_t}\frac{\partial P}{\partial S}\operatorname{var}[dS] \\ &= \frac{S_t}{P_t}\frac{\partial P}{\partial S}\operatorname{var}\left[\frac{dS}{S_t}\right] \end{aligned} \tag{A19}$$

Then it follows that:

$$\begin{aligned} \frac{S_t}{P_t}\frac{\partial P}{\partial S} &= \frac{\operatorname{cov}\left[\frac{dP}{P_t}, \frac{dS}{S_t}\right]}{\operatorname{var}\left[\frac{dS}{S_t}\right]} \\ &= \beta \end{aligned} \tag{A20}$$

Finally, taking the expectation of (A18), we obtain:

$$kdt = \beta\mu dt + r(1-\beta)dt \tag{A21}$$

Q.E.D.

Without the subscripts of t,T we write β over the life of the put options as:

$$\beta = \frac{\operatorname{cov}\left(\frac{P_T}{P_t}, \frac{S_T}{S_t}\right)}{\operatorname{var}\left(\frac{S_T}{S_t}\right)} = \frac{S_t}{P_t}\frac{\operatorname{cov}(P_T, S_T)}{\operatorname{var}(S_T)} \tag{A22}$$

The put option risk-adjusted pricing equation is:

$$\begin{aligned} P_t &= e^{-k(T-t)}E_t[\max\{K - S_T, 0\}] \\ &= e^{-k(T-t)}\left[K\int_0^K \phi(S_T)dS_T - \int_0^K S_T\phi(S_T)dS_T\right] \\ &= e^{-k(T-t)}KN(-h_2) - e^{(\mu-k)(T-t)}S_tN(-h_1) \end{aligned} \tag{A23}$$

where t and T are the current time and maturity time of the option, and K is the strike price of the option and

$$\begin{aligned} h_1 &= \frac{\ln S - \ln K + \left(\mu + ½\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \\ h_2 &= h_1 - \sigma\sqrt{T-t} \end{aligned}$$

A.2.2 Proposition 2 for put options

The β, based on the life of the put option, can be written as:

$$\beta = \frac{S_t}{P_t}\frac{-\left[e^{\sigma^2(T-t)}N(-h_3) - \frac{K}{S_t}e^{-\mu(T-t)}\{N(-h_1) - N(-h_2)\} - N(-h_1)\right]}{e^{\sigma^2(T-t)} - 1} \quad \text{(A24)}$$

Proof:
The expected value of the put payoff using the risk-adjusted measure is:

$$E[P_T] = KN(-h_2) - e^{\mu(T-t)}S_t N(-h_1) \quad \text{(A25)}$$

From the known result of the moment generating function of a Gaussian variable, we have:

$$\begin{aligned} \text{var}[S_T] &= E[S_T^2] - (E[S_T])^2 \\ &= S_t^2 e^{(2\mu+\sigma^2)(T-t)} - S_t^2 e^{2\mu(T-t)} \\ &= S_t^2 e^{2\mu(T-t)}\left(e^{\sigma^2(T-t)} - 1\right) \end{aligned} \quad \text{(A26)}$$

and

$$\begin{aligned} E[S_T P_T] &= \int_0^\infty S_T \max\{K - S_T, 0\}\phi(S_T)dS_T \\ &= K\int_0^K S_T\phi(S_T)dS_T - \int_0^K S_T^2\phi(S_T)dS_T \\ &= KS_t e^{\mu(T-t)}N(-h_1) - S_t^2 e^{(2\mu+\sigma^2)(T-t)}N(-h_3) \end{aligned} \quad \text{(A27)}$$

where

$$h_3 = \frac{\ln S - \ln K + \left(\mu + \frac{3}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Hence, the covariance term in (A22) can be computed as:

$$\begin{aligned} \text{cov}[S_T, P_T] &= E[S_T P_T] - E[S_T]E[P_T] \\ &= [KS_t e^{\mu(T-t)}N(-h_1) - S_t^2 e^{(2\mu+\sigma^2)(T-t)}N(-h_3)] - S_t e^{\mu(T-t)}[KN(-h_2) - S_t e^{\mu(T-t)}N(-h_1)] \\ &= KS_t e^{\mu(T-t)}[N(-h_1) - N(-h_2)] - S_t^2 e^{2\mu(T-t)}[e^{\sigma^2(T-t)}N(-h_3) - N(-h_1)] \\ &= S_t^2 e^{2\mu(T-t)}\left[\frac{K}{S_t}e^{-\mu(T-t)}\{N(-h_1) - N(-h_2)\} - e^{\sigma^2(T-t)}N(-h_3) + N(-h_1)\right] \end{aligned} \quad \text{(A28)}$$

Finally, combining equations (A22), (A26), and (A28) we have:

$$\beta = \frac{S_t}{P_t} \frac{-\left[e^{\sigma^2(T-t)}N(-h_3) - \dfrac{K}{S_t}e^{-\mu(T-t)}\{N(-h_1) - N(-h_2)\} - N(-h_1)\right]}{e^{\sigma^2(T-t)} - 1} \quad \text{(A29)}$$

Q.E.D.

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