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Sensitivity of Systematic Risk Estimates to the Return Measurement Interval Under Serial Correlation

DONGCHEOL KIM
Department of Finance and Economics, Faculty of Management, Rutgers University, Janice Levin Building, New Brunswick, NJ 08903, E-mail: dckim@rci.rutgers.edu

Abstract. This paper analytically and empirically investigates the sensitivity of the return measurement interval to the market beta estimate and suggests a market beta estimation method incorporating the investment horizon through a vector autoregressive (VAR) model when there is serial correlation in returns. The analytical relation between the beta estimate and the return measurement interval is obtained. Based on the analytical relation, a decision function for the intervalling effect is provided. It is found that the intervalling effect is mostly caused by January returns.

Key words: intervalling-effect in beta, serial correlation in returns, vector autoregression

1. Introduction

Estimation of the non-diversifiable, systematic risk, so-called *beta*, is important in the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972), and is used in other settings such as hedging and risk management. Typically, an asset's beta is estimated by the ordinary least squares (OLS) regression model of the rate of returns of the asset on those of the market portfolio.

It is clear that returns measured over shorter periods should have more information on assets' risks than returns measured over longer periods. However, returns measured over shorter periods such as daily suffer from friction in the trading process, known as the nonsynchronous trading problem, which includes serial correlation in the returns. Unfortunately, the OLS beta estimation using serially correlated returns is biased. To overcome this problem, Scholes and Williams (1977), Dimson (1979), and Cohen, Hawawini, Maier, Schwartz, and Whitcomb [hereafter CHMSW] (1983a) have suggested consistent beta estimation methods.[1]

When unit-period returns are subject to serial correlation, beta estimates are not invariant to the return measurement interval or the investment horizon, since the longer-period returns are still subject to the serial correlation of unit-period returns, although the serial correlation becomes weaker as the return measurement interval increases (see Appendix *A* for longer-period returns' serial correlation when unit-period returns follow an autoregressive process of order 1). Put more specifically, estimated beta values are systematically changed as the return measurement interval is varied, which is known as the intervalling effect (see Jensen (1969), Lee (1976a, 1976b), Levhari and Levi (1977), Smith (1978), and Handa, Kothari, CHMSW (1983a, 1983b), and Wasley (1989, 1993)). Even though the CAPM implicity assumes that assets' betas are invariant to the investment horizon, there are some reasons for supposing that betas are sensitive to the investment horizon. First, as mentioned by Handa, Kothari, and Wasley (1993), investors differently perceive risk of an asset according to their investment horizon. As the investment horizon becomes longer, investors in high-risk firms would perceive more risk as opposed to those in low-risk firms. Second, as shown by Levhari and Levy (1977) and Handa, Kothari, and Wasley (1989), the covariance of an asset's (buy-and-hold) return with the market return and the variance of the market return do not change proportionately according to the investment horizon. Third, when the single-period CAPM relation is extended into the H-period ($H > 1$) equilibrium risk-return relation, the single-period beta no longer has a linear relation with the H-period (buy-and-hold) expected return, unless the asset's beta is one. In order to maintain the linear equilibrium relation between betas and expected returns in the H-period investment horizon, a different measure of betas that adjusts for the investment horizon should be used.

The purpose of this paper is to suggest a consistent beta estimator that adjusts for the investment horizon when return observations are serially correlated. This consistent beta estimation takes into account security returns' contemporaneous, lead, and lagged own- and cross-relation (i.e., bivariate serial correlation) structure with returns on the market portfolio through the vector autoregressive (VAR) process. The VAR beta estimator can be represented as an explicit functional form of the investment horizon, and it can therefore adjust for the investment horizon. That is, given a serial bivariate correlation structure, the systematic risk for a specific investment horizon can be estimated, without measuring returns over the corresponding return measurement interval and estimating it through, say, the OLS method. The VAR beta estimation is therefore useful when the available return observations are limited, but the systematic risk for a longer investment horizon (for example, infinite investment horizon) needs to be measured. A decision function for the intervalling effect, that is, the sensitivity of the beta estimate to the return measurement interval, is developed from the VAR beta estimator to determine which correlation structure causes increasing or decreasing beta estimates with the investment horizon. This decision function would be useful to determine the extent of the intervalling effect. Based on the decision function for the intervalling effect, it has been found that the intervalling effect mostly results from January returns.

The rest of the paper is organized as follows: Section 2 presents the VAR beta estimation method. Section 3 presents the results of a Monte Carlo simulation study comparing the estimation efficiency of the VAR method with the OLS, Scholes-Williams, CHMSW, and Dimson methods. Section 4 empirically illustrates the VAR beta estimation and the use of the decision function to determine the intervalling effect, and examines a seasonality in the intervalling effect. Section 5 concludes.

2. A VAR Beta Estimation

In order to derive a consistent beta estimator for the H-period investment horizon or return measurement interval through the VAR process, we assume that:

(A1) Continuously compounded returns (natural logarithm of price relatives) are used in estimating beta.
(A2) Continuously compounded unit-period returns are serially correlated.

Let $R_{it}(H)$ and $R_{mt}(H)$ be the (non-overlapped) H-period continosly compounded returns on an asset i and on the market portfolio over a particular period t, respectively. Then $R_{it}(H) = \sum_{s\in t}^{H} r_{is}$, *and* $R_{mt}(H) = \sum_{s\in t}^{H} r_{ms}$, where r_{is} and r_{ms} are continuously compounded unit-period returns (i.e., the natural logarithm of one plus unit-period buy-and-hold return) on asset i and on the market portfolio at time $s \in t$, respectively. The subscript s indicates unit-period, while the subscript t indicates a longer period. If $t = 1,\ldots,T$, then $s = 1,\ldots,H,H+1,\ldots,2H,2H+1,\ldots,3H,\ldots,(T-1)H+1,\ldots,TH$. For example, when the unit-period is a day (indexed with s), the H-period or t can be a week ($s = 1,\ldots,5(=H)$), a month ($s = 1,\ldots,21(=H)$), or a year ($s = 1,\ldots,250(=H)$).

Proposition 1. *For $H \geq 2$, the beta for the H-period investment horizon, or the H-period beta, of an asset i is defined by*

$$\beta_i(H) = \frac{\beta_i^{(0)} + \sum_{\tau=1}^{H-1}(1-\tau/H)\,\beta_i^{(\tau)} + \sum_{\tau=1}^{H-1}(1-\tau/H)\,\beta_i^{-(\tau)}}{1 + 2\sum_{\tau=1}^{H-1}(1-\tau/H)\,\rho_m(\tau)}, \qquad (1)$$

where

$$\beta_i^{-(\tau)} = \frac{\text{Cov}(r_{is}, r_{ms+\tau})}{\text{Var}(r_{ms})}, \qquad \beta_i^{(\tau)} = \frac{\text{Cov}(r_{is}, r_{ms-\tau})}{\text{Var}(r_{ms})},$$

$$\beta_i^{(0)} = \frac{\text{Cov}(r_{is}, r_{ms})}{\text{Var}(r_{ms})}, \qquad \rho_m(\tau) = \frac{\text{Cov}(r_{ms}, r_{ms-\tau})}{\text{Var}(r_{ms})}. \qquad (2)$$

A proof of Proposition 1 is in Appendix *B*. Note that $\beta_i^{(0)}$, $\beta_i^{-(\tau)}$, and $\beta_i^{(\tau)}$ are the contemporaneous, lead, and lagged unit-period betas, respectively, and $\rho_m(\tau)$ is the τ-th autocorrelation of the unit-period market returns. In particular, $\beta_i^{(0)}$ is the unit-period beta ($= \beta_i(1)$).

If unit-period returns have no serial correlation, the variances and the contemporaneous, lead, and lagged own- and cross-variances in equation (2) can be estimated through ordinary sample moments. In this case, the beta estimate of equation (1) can be approximately transformed into conventional beta estimates. The Scholes-Williams beta (1977) equals $\beta_i(2)$ (two-day beta) if $(1 - \tau/H)$ is set to 1, and the Dimson beta (1979) is the k-period beta of equation (1) by choosing k ($k \ll H$) according to the degree of thinness of asset returns and assuming that market return observations are independent ($\rho_m(\cdot) = 0$). The CHMSW beta (1983a) also equals the k-period beta by choosing k ($k \ll H$) according to the extent of the price-adjustment delays.

However, when unit-period returns are serially correlated, the variance and covariance estimates using ordinary sample moments are not efficient and so is the estimate of the H-period beta, $\hat{\beta}_i(H)$, based on the variance and covariance estimates. In order to efficiently estimate the covariances of equation (2) under serial correlation of unit-period returns, we therefore need a formulation that takes into account the bivariate behavior of unit-period returns on an asset and on the market portfolio, that is, the contemperaneous, lead, and lagged own- and cross-correlation structure. For this, I assume the following return generating process:

(A3) Unit-period (mean-adjusted) returns on an asset i and on the market portfolio over a particular period are generated from the stationary VAR process of order p (VAR(p)) defined as

$$\mathbf{r}_s = \mathbf{\Phi}_1 \mathbf{r}_{s-1} + \ldots + \mathbf{\Phi}_p \mathbf{r}_{s-p} + \epsilon_s, \tag{3}$$

where

$$\mathbf{r}_s = (r_{is}, r_{ms})', \mathbf{\Phi}_p = \begin{pmatrix} \phi_{i1,p} & \phi_{i2,p} \\ \phi_{m1,p} & \phi_{m2,p} \end{pmatrix}, \ \epsilon_s = (\epsilon_{is}, \epsilon_{ms})'.$$

The VAR coefficents $\phi_{i1,p}$ and $\phi_{i2,p}$ relate the current observed return of an asset (r_{is}) to the past observed returns of its own (r_{is-p}) and the market (r_{ms-p}), respectively. These coefficients result from friction in the trading process that causes price-adjustment delays. Therefore, $\phi_{i1,p}$ and $\phi_{i2,p}$ represent the degree of delays of an asset's own information adjustment and the market information adjustment to the current observed return, respectively.

Proposition 2. *If unit-period returns have a serial correlation structure of equation (3), the H-period beta of asset i (H ≥ 2) is defined as*

$$\beta_i(H) = \frac{\beta_i^{(0)} + \sum_{\tau=1}^{H-1} \sum_{k=1}^{p} (1 - \tau/H) \left(\phi_{i1,k}^{[\tau]} \beta_i^{-(k-1)} + \phi_{m2,k}^{[\tau]} \beta_i^{(k-1)} + \dfrac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}} \phi_{i2,k}^{[\tau]} + \dfrac{\sigma_{ii}^{(k-1)}}{\sigma_{mm}^{(0)}} \phi_{m1,k}^{[\tau]} \right)}{1 + 2 \sum_{\tau=1}^{H-1} \sum_{k=1}^{p} (1 - \tau/H) \left(\phi_{m1,k}^{[\tau]} \beta_i^{-(k-1)} + \dfrac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}} \phi_{m2,k}^{[\tau]} \right)}, \tag{4}$$

where

$$\begin{aligned} \beta_i^{(k)} &= \sigma_{im}^{(k)}/\sigma_{mm}^{(0)}, & \beta_i^{-(k)} &= \sigma_{im}^{-(k)}/\sigma_{mm}^{(0)}, \\ \sigma_{im}^{(k)} &= \mathrm{Cov}(r_{is}\, r_{ms-k}), & \sigma_{im}^{-(k)} &= \mathrm{Cov}(r_{is}\, r_{ms+k}), \\ \sigma_{mm}^{(k)} &= \mathrm{Cov}(r_{ms}\, r_{ms-k}), & \sigma_{ii}^{(k)} &= \mathrm{Cov}(r_{is}\, r_{is-k}), \end{aligned} \tag{5}$$

and $\phi^{[\tau]}_{ij,k}$ *is the (i, j)-th element of* $\mathbf{\Phi}^{\tau}_{k}$. *For H=1,* $\beta_i(1) = \beta^{(0)}_i$.

A proof of Proposition 2 is in Appendix *C*. If the VAR order is one ($p = 1$), the *H*-period beta is defined as

$$\beta_i(H) = \frac{\beta^{(0)}_i \{1 + \sum_{\tau=1}^{H-1} (1 - \tau/H)(\phi^{[\tau]}_{i1,1} + \phi^{[\tau]}_{m2,1})\} + \sum_{\tau=1}^{H-1} (1 - \tau/H)[(\sigma^{(0)}_{ii}/\sigma^{(0)}_{mm})\,\phi^{[\tau]}_{m1,1} + \phi^{[\tau]}_{i2,1}]}{1 + 2\beta^{(0)}_i \sum_{\tau=1}^{H-1} (1 - \tau/H)\,\phi^{[\tau]}_{m1,1} + 2\sum_{\tau=1}^{H-1} (1 - \tau/H)\,\phi^{[\tau]}_{m2,1}},$$

where

$$\mathbf{\Phi}^{\tau}_1 = \begin{pmatrix} \phi^{[\tau]}_{i1,1} & \phi^{[\tau]}_{i2,1} \\ \phi^{[\tau]}_{m1,1} & \phi^{[\tau]}_{m2,1} \end{pmatrix}.$$

A consistent estimator of the *H*-period beta of asset *i*, $\hat{\beta}_i(H)$, can be obtained by substituting the unknown parameters in equation (4) with their consistent estimates. The consistent estimates of Φ's are obtained from estimating the VAR equation (3) and those of the lagged ($\sigma^{(k)}_{im}$), lead ($\sigma^{-(k)}_{im}$), and contemporaneous ($\sigma^{(0)}_{im}$) covariances are obtained from equations (A1), (A2), and (A3) in Appendix *C*, respectively. More specifically, in order to obtain the beta estimate for a specific investment horizon, *H*, we first estimate the VAR coefficients by using unit-period return observations (i.e., most frequency data) over a stationary period, and then set the length of the investment horizon. For example, assuming that daily return data is stationary over the previous five years, the beta estimate for monthly investment horizons is obtained through equation (4) by first estimating the VAR coefficients with the previous five year's daily return observations and setting $H = 21$. Moreover, a specific calendar month's beta can also be obtained. For the January 1992 beta, we estimate the VAR coefficients by using only January 1992 daily return observations, and set $H = 21$. This January VAR beta estimate adjusts for a monthly investment horizon; that is, it is the function of the covariance between *monthly* returns on an asset and the market portfolio. However, conventional methods such as the OLS, Scholes-Williams, CHMSW, and Dimson approaches can not obtain an investment horizon-adjusted (monthly) beta estimate, since the conventional beta estimates are a function of the covariance between *daily* returns on the asset and the market portfolio, and thus implicitly assume a daily investment horizon.

Estimation of the VAR coefficients, Φ_k, is crucial in the VAR beta estimation. The VAR coefficients can easily be estimated by maximizing the conditional likelihood function, that is, by a straightforward OLS regression (Hamilton 1994). The VAR order can be determined by a model specification test such as Hosking's (1980) portmanteau test, which is the multivariate version of the Ljung-Box (1978) test, or the likelihood ratio test.

We now address the issue of what happens when we increase the return measurement interval (*H*) to the limit. Since the stationary VAR process has $\Phi^{\tau}_k \rightarrow 0$ ($k = 1,\ldots, p$) for relatively large τ and the variance and covariances are fixed numbers for a certain lag *k,* $\beta_i(H)$ of equation (4) quickly converges to a limiting value, β^*_i. Therefore, we have the following:

Proposition 3. *The H-period beta of an asset i for an infinite investment horizon is given by*

$$\lim_{H \to \infty} \beta_i(H) = \beta_i^*, \tag{6}$$

where β_i^* is a finite limiting number.

A proof of Proposition 3 is omitted. In obtaining the limiting value (i.e., systematic risk for infinte investment horizons), conventional estimation methods are not practical with a finite set of observations, since taking H to the limit is not possible. Due to this problem, CHMSW (1983b) used a two-stage regression method to obtain a limiting value as the asymptote in which the OLS beta estimates of $\beta_i(H)$ for different (finite) H are regressed on an appropriate functin of H. This VAR approach, however, provides a direct estimation of the limiting value.

3. A Monte Carlo Simulation Study

We consider four alternative methods for risk measurement when unit-period (daily) returns are serially correlated. Those are the OLS, Scholes-Williams (1977), Dimson (1979), and CHMSW (1983a) methods.

The OLS beta estimate, $\hat{\beta}_i^{OLS}$, is obtained by estimating the slope coefficient of the market model by the OLS method;

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}, \tag{7}$$

where R_{it} and R_{mt} are returns on asset i and on the market portfolio measured over a return measurement interval, respectively. The Scholes and Williams estimator is a variant of the OLS beta estimate in which the market model is run separately with contemporaneous (i.e., synchronous), lead, or lagged market returns. The consistent estimator is given by the sum of these three separate OLS slope estimates divided by one plus two times the first-order autocorrelation of R_{mt}, $\hat{\rho}_1$, that is,

$$\hat{\beta}_i^{SW} = \frac{\hat{\beta}_i^{-(1)} + \hat{\beta}_i^{(0)} + \hat{\beta}_i^{+(1)}}{1 + 2\,\hat{\rho}_1},$$

where $\hat{\beta}_i^{-(1)}$, $\hat{\beta}_i^{(0)}$, and $\hat{\beta}_i^{+(1)}$ are the OLS estimates from three separate simple regressions of R_{it} on R_{mt+1}, R_{mt}, or R_{mt-1}, respectively. The Dimson estimator is obtained by first regressing R_{it} on lead, synchronous, and lagged values of the market returns to obtain a set of slope coefficients,

$$R_{it} = \alpha_i + \sum_{k=-m}^{m} \beta_{ik} R_{mt+k} + \epsilon_{it}.$$

The Dimson estimator is then obtained by summing the slop coefficients, that is,

$$\hat{\beta}_i^D = \sum_{k=-m}^{m} \hat{\beta}_{ik}.$$

The CHMSW estimator is given by

$$\hat{\beta}_i^{CHMSW} = \frac{\hat{\beta}_i^{(0)} + \sum_{k=1}^{m} \hat{\beta}_i^{-(k)} + \sum_{k=1}^{m} \hat{\beta}_i^{+(k)}}{1 + \sum_{k=1}^{m} \hat{\beta}_m^{-(k)} + \sum_{k=1}^{m} \hat{\beta}_m^{+(k)}},$$

where $\hat{\beta}_i^{-(k)}$, $\hat{\beta}_i^{(0)}$, and $\hat{\beta}_i^{+(k)}$ are the OLS estimates from three separate simple regressions of R_{it} on R_{mt+k}, R_{mt}, or $R_{mt-k,}$, respectively, and $\hat{\beta}_m^{-(k)}$ and $\hat{\beta}_m^{+(k)}$ are the OLS estimates from the separate simple regressions of R_{mt} on R_{mt+k} and R_{mt-k}, respectively.

Since it is difficult to analytically compare the statistical efficiency of the VAR beta estimate with the four alternative methods given above, we perform a Monte Carlo simulation. To simulate each firm's 1,260 daily returns (i.e., 60 months) as unit-period returns through the market model of equation (7), the market returns and the idiosyncratic returns ϵ_{it} of each firm are needed. 1,260 CRSP (Center for Research in Security Prices at the University of Chicago) equal-weighted daily market returns from 1988 to 1992 are used as the true market returns, and ϵ_{it} is simulated by assuming an AR(1) process. The parameters of the AR(1) process for simulating ϵ_{it} are obtained as follows: we first estimate the market model using actual daily returns of the 2,550 firms whose daily data are available for at least one year over the period from 1988 to 1992 from the CRSP daily return tape, and then estimate the AR(1) parameters of the residual returns. These estimated AR(1) parameters of the 2,550 firms are used as the true AR(1) coefficients for simulating ϵ_{it} in equation (7). All stocks are assigned a 'true' beta in equation (7) of 1.0, and the alpha of equation (7) is determined according to their average returns. Sixty *monthly* continuously compounding returns are then computed, and beta is estimated through the OLS, Scholes-Williams, Dimson with $m = 2$ (2 lags and 2 lead), and CHMSW (with $m = 2$) methods. For the VAR method, the VAR order should be determined first. By the Hosking's (1980) protmanteau test and the likelihood ratio test, the VAR(1) is found to be most appropriate. Then the VAR coefficients are estimated using the 1,260 daily simulated returns, and the VAR beta estimate is obtained from equation (4) by setting $H = 21$. All beta estimates therefore adjust for monthly investment horizon.

Table 1 presents the results of 100 simulations: the average beta estimates, the biases, and the mean squared errors (MSE) of each estimation method in each decile (based on the estimated betas). The spread of beta estimates over deciles is the lowest in the VAR method and the greatest in the Dimson method. To be more specific, the MSE of the VAR beta estimates is 0.05, while the MSEs of the OLS, Scholes-Williams, CHMSW, and Dimson estimates are 0.174, 0.208, 0.390, and 0.605, respectively. The VAR beta estimate also has the lowest bias. In particular, the CHMSW and the Dimson methods could give greatly underestimated (even negative) or overestimated beta values. The above simulation

Table 1. Average beta estimates† using monthly continuous compounding returns based on the simulated 1260 daily returns of 2,550 firms when all true betas are one.

		Beta estimation methods			
Decile of estimated betas	OLS	Scholes-Williams	CHMSW	Dimson	VAR
1	0.257	0.080	−0.116	−0.378	0.619
2	0.706	0.515	0.544	0.449	0.852
3	0.827	0.633	0.733	0.679	0.914
4	0.903	0.709	0.853	0.822	0.954
5	0.964	0.770	0.950	0.939	0.985
6	1.021	0.827	1.040	1.046	1.015
7	1.081	0.887	1.136	1.162	1.047
8	1.155	0.960	1.253	1.300	1.086
9	1.269	1.072	1.435	1.521	1.147
10	1.703	1.495	2.105	2.349	1.389
Ave	0.989	0.795	0.993	0.989	1.001
Bias††	−0.011	−0.205	−0.007	−0.011	0.001
MSE††	0.174	0.208	0.390	0.605	0.050

†The numbers in the table are the average values of 20 simulations.
††*Bias* $= \sum_{i=1}^{N} (\hat{\beta} - \beta_0)/N$, and *MSE* $= \sum_{i=1}^{N} (\hat{\beta}_i - \beta_0)^2/N$, where $\hat{\beta}_i$ is the estimated beta of a firm i, β^0 is the true beta value of 1.0, and N is the total number firms in the simulation (= 2,550).

has also been repeated with different simluation settings: the other same true beta values of 1.5, 0.5, or all different true beta values for all firms, the CRSP value-weighted market returns, different return measurement intervals, and/or independent ϵ_{it}. However, the results (not reported) are similar; the VAR beta estimator is the most robust and efficient.

4. The Intervalling Effect and VAR Beta Estimation

4.1 A Decision Function for the Intervalling Effect

In fact, the VAR beta estimate of equation (4) represents an explicit functional form relating beta estimate and the investment horizon, H. Based on this functional form, we can determine which serial correlation structure induces beta to increase or decrease with the investment horizon. Sacrificing some accuracy, we obtain the following approximate decision function for the intervalling effect:

Proposition 4. *The decision function for the intervalling effect of an asset i under VAR(p) is defined as*

$$C_i = \beta_i^{(0)} - 1 + \sum_{k=1}^{p} (\phi_{i1,k} - \phi_{m1,k})\beta_i^{-(k-1)} + \sum_{k=1}^{p} (\phi_{i2,k} - \phi_{m2,k}) \frac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}}$$
$$- \sum_{k=1}^{p} \phi_{m1,k} \left(\beta_i^{-(k-1)} - \frac{\sigma_{ii}^{(k-1)}}{\sigma_{mm}^{(0)}} \right) + \sum_{k=1}^{p} \phi_{m2,k} \left(\beta_i^{(k-1)} - \frac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}} \right).$$

If the value of the decision function C_i is positive (negative), then the beta estimate increases (decreases) with the return measurement interval. The greater the positive (negative) value of C_i, the more rapidly the beta estimate increases (decreases) with the return measurement interval. For a small order VAR, this decision function can be approximated as

$$C_i \approx \sum_{k=1}^{p} \phi_{i1,k} - \sum_{k=1}^{p} \phi_{m2,k} + \sum_{k=1}^{p} \phi_{i2,k} - \left(2 - \frac{\sigma_i^2}{\sigma_m^2} \right) \sum_{k=1}^{p} \phi_{m1,k} \,, \tag{8}$$

where σ_i^2 and σ_m^2 are the variances of unit-period returns on the asset i and the market portfolio, respectively.

A proof of Proposition 4 is in Appendix *D*. I will address the intervalling effect based on the approximate decision function of equation (8) since this expression is more tractable and intuitive. Proposition 4 indicates that the beta estimate of an asset increases more rapidly with the return measurment interval, (i) as the asset's returns are more autocorrelated (measured by $\sum_{k=1}^{p} \phi_{i1,k}$) and more correlated with the lagged market returns (measured by $\sum_{k=1}^{p} \phi_{i2,k}$), (ii) as the sums of the coefficients of the market returns to the asset's own and to its asset's lagged returns ($\sum_{k=1}^{p} \phi_{m2,k}$ and $\sum_{k=1}^{p} \phi_{m1,k}$, respectively) are smaller, and (iii) as the asset's own variance is greater than the market variance when $\sum_{k=1}^{p} \phi_{m1,\,k}$ is positive. Put differently, the beta estimates increase with the return measurement interval if the asset's unit-period returns are more serially correlated than are the unit-period returns on the market, that is, $\sum_{k=1}^{p}\phi_{i1,k} + \sum_{k=1}^{p} \phi_{i2,k} > \sum_{k=1}^{p} \phi_{m2,k} + \sum_{k=1}^{p} \phi_{m1,k}$, and vice versa. Note that for most assets, σ_i^2/σ_m^2 is less than 2.

4.2 An Empirical Illustration of the Decision Criterion for the Intervalling Effect

To empirically illustrate the usefulness of the decision function for the intervalling effect and examine whether there is seasonality in the intervalling effect, we construct 20 size portfolios using daily returns over the period from January 1963 to December 1992 of all common stocks (NYSE and AMEX) listed on the CRSP daily tape as follows: at the end of each year, all available stocks are ranked by their market value (number of shares outstanding times price per share) in December and are assigned into one of 20 portfolios based on their relative position in terms of market value. The compositions of the size-sorted portfolios are rebalanced annually. The post-ranking portfolio returns are computed by combining daily returns of all individual securities within the portfolios with equal weights. The CRSP equally-weighted market returns are used as the returns on the market portfolio.

By the multivariate portmanteau test (Hosking, 1980) and the likelihood ratio test (testing results are not reported here), the order of five for the VAR process (VAR(5)) is found to be most appropriate for the daily returns of the 20 size portfolios over the whole 30-year period. Table 2 presents the VAR beta estimates of the size-sorted portfolios with return measurement intervals of one day ($H = 1$), one month ($H = 21$), one year ($H = 251$), and infinitely ($H = \infty$). The first 10 small-size portfolios' beta estimates monotonically increase with the return measurment interval, but the last 10 large-size portfolios' beta estimates monotonically decrease. The increasing (decreasing) rate is larger as the market value is smaller (larger). Notably, the mid-size portfolios' beta estimates barely change.

The last column in Table 2 shows each size portfolio's decision function C_i values of equation (8). The small-size portfolios (Portfolio 1 through 10) have positive C_i values, while the large-size portfolios (Portfolio 11 through 20) have negative C_i values. The smaller (larger) the firm size, the greater the positive (negative) C_i values. These estimated C_i values imply that small-size portfolios have increasing betas, while large-size portfolios have decreasing betas with the return measurement interval. This implication is supported

Table 2. H-period beta estimates ($\hat{\beta}_i(H)$) of 20 equally-weighted size-sorted portfolios with the CRSP equal-weighted market returns through a vector autoregressive process of order 5 (VAR(5)) using daily returns over the period January 1963 to December 1992.

Size portfolio	Return Measurement Interval (H): one-day $\hat{\beta}_i(1)$	one-month $\hat{\beta}_i(21)$	one-year $\hat{\beta}_i(251)$	∞ $\hat{\beta}_i(\infty)$	$\hat{\beta}_i(\infty) - \hat{\beta}_i(1)$	$\hat{C}_i$ values
smallest	0.9082	1.2842	1.3597	1.3657	0.4111	0.3385
2	0.9452	1.1739	1.2036	1.2060	0.2354	0.2263
3	0.9603	1.1293	1.1488	1.1503	0.1860	0.1885
4	0.9854	1.1199	1.1378	1.1392	0.1400	0.1607
5	1.0005	1.1134	1.1253	1.1262	0.1213	0.1473
6	1.0122	1.0634	1.0682	1.0686	0.0578	0.0682
7	1.0204	1.0665	1.0706	1.0709	0.0464	0.0613
8	1.0249	1.0591	1.0619	1.0622	0.0364	0.0576
9	1.0532	1.0599	1.0612	1.0613	0.0103	0.0368
10	1.0438	1.0568	1.0583	1.0584	0.0149	0.0458
11	1.0437	1.0371	1.0367	1.0367	−0.0097	0.0271
12	1.0403	1.0005	0.9958	0.9954	−0.0355	−0.0238
13	1.0173	0.9817	0.9755	0.9750	−0.0427	−0.0309
14	0.9967	0.9443	0.9373	0.9367	−0.0492	−0.0571
15	0.9934	0.9120	0.9010	0.9001	−0.0895	−0.0877
16	1.0113	0.9038	0.8874	0.8861	−0.1163	−0.0995
17	0.9838	0.8636	0.8453	0.8439	−0.1374	−0.1178
18	0.9584	0.8009	0.7795	0.7778	−0.1744	−0.1714
19	0.9565	0.7565	0.7306	0.7285	−0.2205	−0.2211
largest	0.9344	0.6502	0.6161	0.6135	−0.3153	−0.3378

from the actual beta estimates. That is, the magnitude of the beta estimates' increasing or decreasing rates is consistent with the magnitude of C_i values. Therefore, the approximated decision function C_i could provide useful guidance for deciding which asset's returns have increasing or decreasing beta estimates with various investment horizons, without explicitly examining the trend of beta estimates over various return measurement intervals.

4.3 Seasonality in the Intervalling Effect

The decision function C_i would also be useful to examine whether there is seasonality, especially a January effect, in the intervalling effect, since it is hard to explicitly examine the trend of beta estimates over various return measurement intervals by using only January returns.

In order to examine whether there is a January effect in the intervalling effect, we estimate the C_i values of the 20 size-sorted portfolios using January and non-January months' returns in each year. The average C_i values are presented in Table 3. The average C_i values of non-January months are small and have no distinctive trend across portfolio size. On the other hand, the average C_i values of January are relatively larger and in a similar pattern to those of the whole sample in Table 2. The smaller (larger) the firm size, the greater the January positive (negative) C_i values. Furthermore, they are more widely spread across portfolio size than are the C_i values of whole sample. These C_i values imply that the intervalling effect mostly results from January returns.

In order to confirm the implication of the non-January months' C_i values, we partition a calendar year into 12 one-month, 6 two-month, 4 three-month, 3 four-month, and 2 six-month periods, exclude the period of the year containing January (i.e., the first period of the year in each partition), and estimate betas by the OLS method using return observations of one-month, two-month, three-month, four-month, and six-month return measurment intervals. Table 3 also presents the January-excluded OLS beta estimates of the size-sorted portfolios over those return measurement intervals. As we see from Table 3, the previously known intervally effect is hardly found when January returns are excluded. There is almost no distinctive trend in the beta estimates across the return measurement interval, which is consistent with the non-January months' C_i values. It can be asserted, therefore, that the intervalling effect mostly results from January returns.

5. Conclusion

This paper have suggested a consistent beta estimation method that adjusts for a particular investment horizon. This estimation method captures the bivariate serial correlation structure of unit-period returns on an asset and on the market through the VAR process. A Monte Carlo simulation study shows that this VAR beta estimator is robust and has better statistical efficiency than conventional methods such as the OLS, Scholes-Williams, CHMSW, and Dimson estimators. A decision function for the intervalling effect has also

Table 3. OLS beta estimates when the period of the year containing January is excluded, and average values of the decision function C_i for the intervalling effect of January and non-January months over the period January 1963 to December 1992.

	OLS beta estimates† (with January-exluded period's returns)					Average $\hat{C}_i$ values††	
Size portfolio	one-month	two-month	three-month	four-month	six-month	Non-January	January
smallest	1.2071	1.1586	1.2210	1.2200	1.2188	−0.0726	0.4524
2	1.1503	1.1684	1.1693	1.1739	1.1843	0.0013	0.3821
3	1.1303	1.1468	1.1384	1.1399	1.2092	−0.0177	0.2115
4	1.1097	1.1314	1.1270	1.1241	1.1259	−0.0209	0.1364
5	1.1144	1.1140	1.1230	1.1213	1.1227	−0.0100	0.0601
6	1.0791	1.0926	1.0792	1.1230	1.1204	0.0065	0.0747
7	1.0735	1.1031	1.0696	1.1066	1.1063	0.0066	0.0632
8	1.0721	1.0845	1.0663	1.0775	1.0760	−0.0002	0.0592
9	1.0702	1.0828	1.0775	1.0666	1.0772	0.0080	0.0561
10	1.0547	1.0707	1.0634	1.0453	1.0896	0.0193	0.0656
11	1.0335	1.0498	1.0311	1.0084	0.9990	0.0531	0.0408
12	1.0094	1.0269	1.0020	1.0115	1.0480	0.0455	−0.0314
13	0.9822	0.9869	0.9845	0.9793	0.9511	0.0534	−0.0662
14	0.9600	0.9689	0.9645	0.9604	0.9688	0.0502	−0.0685
15	0.9108	0.9188	0.8925	0.8585	0.8519	0.0493	−0.0768
16	0.9119	0.8985	0.9135	0.8108	0.7979	0.0690	−0.0857
17	0.8696	0.8498	0.8646	0.8002	0.7504	0.0414	−0.1006
18	0.8071	0.7931	0.8124	0.7256	0.7200	−0.0134	−0.1346
19	0.7519	0.7162	0.7434	0.6740	0.6749	−0.0073	−0.1704
largest	0.6572	0.6054	0.6318	0.5802	0.5831	−0.0726	−0.2500

†After partitioning a calendar year into 12 one-month, 6 two-month, 4 three-month, 3 four-month, and 2 six-month periods and excluding the period of the year containing January, we estimate beta by the OLS method using return measurement intervals of one month, two months, three months, four months, and six months.
†C_i is estimated over January and non-January months each year, and we compute the average of each portfolio.

been suggested to determine which asset's beta increases or decreases with the investment horizon and which asset's beta changes more rapidly. This decision function can also be applied to examine whether there is seasonality in the intervalling effect. Based on this decision function, it has been found that January returns cause most of the intervalling effect.

Appendix

A. First-Order Autocorrelation of H-period Returns

Let the unit-period return of an asset, r_s, follow the autoregressive process of order 1 (AR(1)):

$$r_s = \phi r_{s-1} + \epsilon_s.$$

Then the first-order autocorrelation of the H-period continuously compounded return is given by

$$\rho_H(1) = \phi^H + (1 - \phi^{2H})\left(\frac{1}{(Z_1(H) - 2Z_2(H))/Z_2(H) + 2\phi^H}\right),$$

where

$$Z_1(H) = \frac{H(1 - \phi^H)^2}{(1 - \phi)^2},$$

$$Z_2(H) = -\frac{1}{(1 - \phi)^2}\left(H\phi^H - (1 + \phi^H)\frac{(\phi - \phi^{H+1})}{(1 - \phi)} + \frac{(\phi^2 - \phi^{2H+2})}{(1 - \phi^2)}\right).$$

For a relatively large value of H, $_{\rho H}(1)$ approaches $\phi\,/[H(1 - \phi^2) - 2\phi]$.

B. Proof of Proposition 1

By definition of the market beta for the H-period investment horizon,

$$\beta_i(H) = \frac{\mathrm{Cov}(R_{it}(H), R_{mt}(H))}{\mathrm{Var}(R_{mt}(H))}$$

$$= \frac{H\,\mathrm{Cov}(r_{is}, r_{ms}) + \sum_{\tau=1}^{H-1} (H - \tau)\,\mathrm{Cov}(r_{is}, r_{ms+\tau}) + \sum_{\tau=1}^{H-1} (H - \tau)\,\mathrm{Cov}(r_{is}, r_{ms-\tau})}{H\,\mathrm{Var}(r_{ms}) + 2\sum_{\tau=1}^{H-1} (n - \tau)\,\mathrm{Cov}(r_{ms}, r_{ms-\tau})},$$

where the subscript s indicates unit-period (for example, one day) and the subscript t indicates a longer period (for example, one week ($H = 5$) or one month ($H = 21$)). Dividing the numerator and denominator of the above equation by $H\,\mathrm{Var}(r_{ms})$, we obtain equation (1).

C. Proof of Proposition 2

Since a higher-order VAR can always be rewritten as a first-order VAR as in Campbell and Shiller (1988), we can obtain the following VAR(1) form, equivalent to the VAR(p), as

$$\mathbf{X}_s = \Phi\,\mathbf{X}_{s-1} + \mathbf{u}_s,$$

where

$$\mathbf{X}_s = (r_{is}, \ldots, r_{is-(p-1)}, r_{ms}, \ldots, r_{mt-(p-1)})', \quad (2p \times 1)$$

$$\boldsymbol{\Phi} = \begin{pmatrix} \phi_{i1,1} & \phi_{i1,2} & \ldots & \phi_{i1,p} & \phi_{i2,1} & \phi_{i2,2} & \ldots & \phi_{i2,p} \\ 0 & 1 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & \ldots & 1 & 0 & 0 & \ldots & 0 \\ & & & & & & & \\ \phi_{m1,1} & \phi_{m1,2} & \ldots & \phi_{m1,p} & \phi_{m2,1} & \phi_{m2,2} & \ldots & \phi_{m2,p} \\ 0 & 0 & \ldots & 0 & 0 & 1 & \ldots & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 1 \end{pmatrix}, \quad (2p \times 2p)$$

$$\mathbf{u}_s = (\epsilon_{is}, 0, \ldots, 0, \epsilon_{ms}, 0, \ldots, 0)'. \qquad (2p \times 1)$$

The lagged, lead, and contemporaneous covariance matrices of $\mathbf{X}_s$ can be represented, respectively, as

$$\Gamma(\tau) = E\,(\mathbf{X}_s \mathbf{X}_{s-\tau}^{'}) = \Phi^{\tau}\Gamma(0), \quad \tau = 0, \ldots, n-1, \tag{A1}$$

$$\Gamma(-\tau) = \Gamma'(\tau), \tag{A2}$$

$$\mathrm{vec}(\Gamma(0)) = (I - \Phi \otimes \Phi)^{-1}\,\mathrm{vec}(\Sigma), \tag{A3}$$

where $\Sigma = \mathrm{Cov}(\mathbf{u}_s)$. Hence, the elements of the $\Gamma(0)$ matrix are estimated through (A3).

Let $\gamma_{j,k}(\tau)$, $\gamma_{j,k}(-\tau)$, and $\gamma_{j,k}(0)$ be the (j, k)-th elements of the covariance matrices $\Gamma(\tau)$, $\Gamma(-\tau)$, and $\Gamma(0)$, respectively ($j, k = 1,\ldots,2, p$). The H-period beta can be obtained by replacing the covariance terms in (1) with these elements. Therefore, the consistent beta estimate for the H-period investment horizon is

$$\hat{\beta}_i(H) = \frac{\hat{\gamma}_{1,p+1}(0) + \sum_{\tau=1}^{H-1}(1-\tau/H)\,(\hat{\gamma}_{1,p+1}(-\tau) + \hat{\gamma}_{1,p+1}(\tau))}{\hat{\gamma}_{p+1,p+1}(0) + 2\sum_{\tau=1}^{H-1}(1-\tau/H)\,\hat{\gamma}_{p+1,p+1}(\tau)}$$

$$= \frac{\hat{\beta}_i^{(0)} + \sum_{\tau=1}^{H-1}\sum_{k=1}^{p}(1-\tau/H)\left(\hat{\phi}_{i1,k}^{(\tau)}\,\hat{\beta}_i^{-(k-1)} + \hat{\phi}_{m2,k}^{(\tau)}\,\hat{\beta}_i^{(k-1)} + \dfrac{\hat{\sigma}_{mm}^{(k-1)}}{\hat{\sigma}_{mm}^{(0)}}\,\hat{\phi}_{i2,k}^{(\tau)} + \dfrac{\hat{\sigma}_{ii}^{(k-1)}}{\hat{\sigma}_{mm}^{(0)}}\,\hat{\phi}_{m1,k}^{(\tau)}\right)}{1 + 2\sum_{\tau=1}^{H-1}\sum_{k=1}^{p}(1-\tau/H)\left(\hat{\phi}_{m1,k}^{(\tau)}\,\hat{\beta}_i^{-(k-1)} + \dfrac{\hat{\sigma}_{mm}^{(k-1)}}{\hat{\sigma}_{mm}^{(0)}}\,\hat{\phi}_{m2,k}^{(\tau)}\right)}.$$

As long as the VAR coefficient estimates are consistent, the VAR beta estimate is also consistent.

D. Proof of Proposition 4

In order to obtain a tractable decision criterion of the intervalling effect, $\beta_i(H)$ of (4) should be simplified. By approximating $\Phi^\tau \to 0$ for $\tau \geq 2$,

$$\beta_i(H) \approx \frac{H\,\beta_i^{(0)} + (H-1)\sum_{k=1}^{p}\left(\phi_{i1,k}\,\beta_i^{-(k-1)} + \phi_{m2,k}\,\beta_i^{(k-1)} + \frac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}}\,\phi_{i2,k} + \frac{\sigma_{ii}^{(k-1)}}{\sigma_{mm}^{(0)}}\,\phi_{m1,k}\right)}{H + 2\,(H-1)\sum_{k=1}^{p}\left(\phi_{m1,k}\,\beta_i^{-(k-1)} + \frac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}}\,\phi_{m2,k}\right)}. \tag{A4}$$

It is clear that the H-period beta estimate increases with the return measurement interval, H, if the following function (i.e., the coefficient of the term H in the numerator minus the coefficient of the term H in the denominator in equation (A4)) is positive, and vice versa:

$$C_i = \beta_i^{(0)} - 1 + \sum_{k=1}^{p}(\phi_{i1,k} - \phi_{m1,k})\beta_i^{-(k-1)} + \sum_{k=1}^{p}(\phi_{i2,k} - \phi_{m2,k})\frac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}}$$
$$- \sum_{k=1}^{p}\phi_{m1,k}\left(\beta_i^{-(k-1)} - \frac{\sigma_{ii}^{(k-1)}}{\sigma_{mm}^{(0)}}\right) + \sum_{k=1}^{p}\phi_{m2,k}\left(\beta_i^{(k-1)} - \frac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}}\right).$$

In order to obtain a more tractable relation between the decision function C_i and the VAR coefficients by sacrificing some accuracy, we roughly approximate

$$\beta_i^{(k-1)} \approx \beta_i^{(0)}, \quad \beta_i^{-(k-1)} \approx \beta_i^{(0)}, \quad \frac{\sigma_{mm}^{(k-1)}}{\sigma_{mm}^{(0)}} \approx 1, \quad \frac{\sigma_{ii}^{(k-1)}}{\sigma_{mm}^{(0)}} \approx \frac{\sigma_i^2}{\sigma_m^2}, \quad k = 1, \ldots, p.$$

Because of the principle of parsimony, the order of the VAR model, p, is usually small. For example, VAR(1) is the most popular model specification and all VAR(p) models can be equivalently expressed into VAR(1) as in Campbell and Shiller (1988). Therefore, the above approximations would roughly hold only for small order of the VAR model. Then I obtain the following approximated decision function:

$$C_i \approx \sum_{k=1}^{p}\phi_{i1,k} - \sum_{k=1}^{p}\phi_{m2,k} + \sum_{k=1}^{p}\phi_{i2,k} - \left(2 - \frac{\sigma_i^2}{\sigma_m^2}\right)\sum_{k=1}^{p}\phi_{m1,k}.$$

Notes

1. Unfortunately, Dimson's estimator is inconsistent (see Fowler and Rorke 1983).

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