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Structural change and time dependence in models of stock returns

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Abstract

In this paper, we provide evidence that the time-series properties of stock returns include both structural change and time dependence in the conditional variance. The absence of a structural change component tends to overstate the persistence parameter in a time-dependent model specification. The reason is that time-dependent model specifications do not distinguish between positive residuals that increase volatility from those that represent a resolution of uncertainty. A sequential mixture of normal distributions model of structural change is employed to estimate discrete change points in the time-series of volatility in either direction. Although the model of structural change appears to be the more descriptive process in a mutually exclusive comparison, a joint model of time dependence and structural change is most likely.

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1. Introduction

The model specification for the statistical distribution of stock returns is crucial for hypothesis testing in empirical research. Dating back to the seminal works of

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Fama (1965) and Mandelbrot (1963), the unconditional distribution of stock returns contained evidence of leptokurtosis, skewness, and volatility clustering. Models of time-varying conditional moments are a natural way of modeling these empirical observations. The recent literature employing the autoregressive conditional heteroskedastic (ARCH) model introduced by Engle (1982) and the generalized ARCH (GARCH) model of Bollerslev (1986) is very appealing. [1] These models are consistent with the notion that conditional moments change at each point in time in response to changes in the information set.

In ARCH, the conditional error distribution is normal with conditional variance equal to a linear function of past squared errors. The GARCH specification allows the current conditional variance to be a function of past conditional variances as well. Since financial theory suggests that an increase in variance (risk proxy) will result in a higher expected return, ARCH and GARCH in the mean models proposed by Engle et al. (1987) and Bollerslev et al. (1988) are also natural extensions. Furthermore, Kim and Kon (1994) provide evidence that intertemporal dependence models for conditional heteroscedasticity with a leverage (or asymmetry) effect are superior to a wide variety of econometric model specifications in the literature. In particular, the Glosten et al. (1993) specification is the most descriptive for individual stocks while Nelson's (Nelson, 1989, 1991) exponential model is the most likely for stock indexes. For empirical studies that will likely choose one model specification for stocks and indexes, the evidence in Kim and Kon (1994) indicates that the Glosten, Jagannathan, and Runkle (GJR) model is the best alternative. Hence, the variant of the GJR model used by Kim and Kon (1994) will be the benchmark in this study.

There is also empirical evidence on the probable existence of structural change in risk that is not accommodated by the intertemporal dependence model specifications. Public announcements of corporate investment and financial decisions that imply a change in the firm's expected return and risk will be impounded in stock prices immediately in an efficient market. The announcements of relevant macroeconomic information will affect the return and risk of all securities, and hence, portfolios (indexes). Since relevant information that changes the risk structure is randomly released with some time interval (not at every moment) in sequence, these information events translate into *sequential* discrete structural shifts (or change-points) for the mean and/or variance parameter(s) in the time-series of security returns. [2] Kim and Kon (1996) provide evidence that the specification of

[1] For example, see Bollerslev (1987), French et al. (1987), Chou (1988), Akgiray (1989), Connolly (1989), Ballie and DeGennaro (1990), Lamoureux and Lastrapes (1990), Pagan and Schwert (1990), Schwert and Seguin (1990), Bodurtha and Mark (1991) and Engle and Ng (1993).

[2] For evidence of parameter shifts, see Beaver (1968), Black (1976), Officer (1972), Boness et al. (1974), Hsu et al. (1974), Patell and Wolfson (1981), Christie (1982), Kon (1984), Pagan and Schwert (1990), Schwert (1989, 1990), Bodurtha and Mark (1991), Haugen et al. (1991), and Kim (1998).

the deterministic sequential discrete structural shifts is much more descriptive of stock returns than other time-independent models in the literature.

Lamoureux and Lastrapes (1990) investigate the extent to which persistence in variance as measured by the GARCH model may be overstated by the failure of taking into account of deterministic structural shifts in the model. They assume that structural shifts in unconditional variance occur at every 302 daily return observations and use arbitrary dummy variables for the structural shifts to show that persistence in variance can be overstated. After allowing for the structural shifts, they find a decline in measured persistence in variance. Their simulations also provide evidence of greater overstatement of persistence in variance when structural shifts are ignored. They conclude that discrete shifts in unconditional variance are a type of persistence in variance whereby shocks contain no information about future evolution. This is in sharp contrast to the predictable volatility changes in the time-dependent models of stock returns.

The purpose of this paper is to document the relative importance of structural change and time dependence for the model specification of the distribution of stock returns. In addition, we provide a procedure to accommodate both structural change in volatility and time dependence in conditional variances of stock returns in one model specification by developing a means of identifying the timing of discrete structural changes. The remainder of this paper is organized as follows: Section 2 provides a model specification and estimation procedure for structural change. Section 3 introduces the procedure to accommodate structural change as an exogenous variable in a GJR-M model specification. Then an analysis of the contribution of time dependence and structural change can be made in a nested model specification. Common features in structural changes are examined in Section 4. The summary and implications are given in Section 5.

2. A model of structural change

Our main objective is to investigate whether persistence in volatility can be captured by including the deterministic structural shifts of the variance parameter rather than simply using arbitrary dummy variables as in Lamoureux and Lastrapes (1990). [3] A deterministic sequential mixture (SM) of normal distributions model specification is defined as a sequence of stock returns $(R_1, R_2, \ldots, R_T)$ that are normal variates with constant variance parameter up to a change-point. Define these as τ_1 return observations from regime 1. Thereafter, return $\tau_1 + 1$ to τ_2 are a sequence of return observations from regime 2, and so forth, until the last sequence of return observations from the Kth regime, $\tau_{K-1} + 1$ to τ_K. For each regime k, the residual or unanticipated return, ϵ_{kt}, is identically and independently normally distributed with mean 0 and variance σ_k^2 for each $t \in [\tau_{k-1} + 1, \tau_k]$. Therefore, parameters $\tau_1, \ldots, \tau_{K-1}$ define the structural change-points.

[3] We tested for very rarely detected mean change-points.

2.1. A detection method of structural variance change-points

The crucial issue is how to effectively and accurately detect the structural change-points in variance. To do this, we employ a statistical detection procedure rather than using the announced calendar dates of economic events. In fact, the use of the announced calendar dates has several practical problems for detecting the change-points. First, in practice, it is difficult to identify all announcement dates over a long period of time that relevant information is released. Even if possible, it would be hard to determine which released information is significant enough to change the risk structure. Some information might have too weak an impact on stock prices to change the risk structure. Moreover, there are some events that are not publicly announced, but can affect the risk structure of firms.

Another problem for the use of announced calendar dates is whether the announcement dates are the same as the true economic event dates at which the structural shift actually occurs; that is, whether the market is informationally efficient. In efficient markets, actual economic event dates should be identical to announced event dates. However, if the relevant information is leaked before the announcement or if the market has a delayed reaction from investors, then actual event dates might be different from announced event dates. In this case, actual event dates could be before or after announcement dates according to information leakage or investors' delayed response. Under these circumstances, therefore, a statistical procedure would be more effective to detect the actual structural shift points.

There are two statistical ways to detect the change points, *simultaneously* and *sequentially*. If the number of regimes, K, is known, the simultaneous detection method selects the set of change points that has the greatest posterior probability (or the likelihood value after adjusting for the number of estimated parameters). In reality, however, the number of regimes is not explicitly known. Furthermore, when the sample size and the number of regimes are relatively large, the simultaneous detection of change-points is computationally very difficult, since all possible partitions should be considered. [4] The alternative we employ is a sequential detection procedure. This methodology is computationally more efficient and can also estimate the unknown number of change-points.

The sequential change-point detection procedure begins with a test of the stationarity of the variance parameter applied to an initial sample by considering the following null hypothesis:

$$H_0: \rho = \sigma_2^2/\sigma_1^2 = 1$$

where σ_1^2 and σ_2^2 are variance parameters of two implicit regimes. The test requires computation of the unconditional p-value for the null hypothesis. This

[4] For example, 7800 return observations and 60 regimes would require approximately 3.2×10^{151} computations of the posterior probability. This situation is typical for the stock return data in this study.

stationarity test of the variance parameter is repeated as each data point is added to the initial sample until the unconditional p-value is less than an assigned significance level (the null hypothesis is rejected). If the null hypothesis is rejected, we assume there is a shift in the variance parameter over that sample. Then we calculate the posterior probability of *each possible* two-regime classification for that sample. The classification having the greatest posterior probability of a change-point is selected as the posteriori most likely change-point for the variance (i.e., generalized maximum likelihood estimator [5]). The first detected change-point for the variance is $\hat{\tau}_1$. We repeat the above procedure with an initial sample beginning at the first data point following the previously detected change-point (i.e., $\hat{\tau}_1 + 1$). When the null H_0 is rejected, we again calculate the posterior probability of each data point over the sample period beginning at $t = \hat{\tau}_1 + 1$. The two-regime classification having the greatest posterior probability of a change-point is selected as the second change-point for the variance, $\hat{\tau}_2$. This procedure is repeated until all data (T observations) are scanned. Then all change-points for the variance have been *sequentially* detected.

The sequential detection of change-points methodology requires both a computational method for the unconditional p-values and the posterior distribution of a change-point, τ, in each test period. We calculate the unconditional p-values based on the highest posterior density (HPD) interval. This is a Bayesian significance test with a diffuse prior (see, in detail, Lindley, 1965; Box and Tiao, 1973; Kim, 1991). This HPD interval test can be used when the information on target parameters is diffuse, while the traditional Bayes test using a posterior odds cannot. Since we have no information on the change of variance parameters of stock returns a priori, the HPD interval test is appropriate. [6]

The unconditional p-value for H_0 by the HPD interval test for sample size n is computed as

$$p_{\rho=1} = \sum_{\tau} 2\{1 - \Upsilon_{\tau-1,n-\tau-1}(F(1))\}\pi(\tau|R), \tag{1}$$

where $\Upsilon_{\tau-1,\,n-\tau-1}(\cdot)$ is the cumulative density function of an F distribution with $(\tau-1,\ n-\tau-1)$ degrees of freedom, and

$$F(\rho) = \max\left\{\rho\left(\hat{\sigma}_1^2/\hat{\sigma}_2^2\right), \frac{1}{\rho}\left(\hat{\sigma}_2^2/\hat{\sigma}_1^2\right)\right\}.$$

[5] See Degroot (1970, p. 236) for further explanation about the generalized maximum likelihood estimator.

[6] Although the HPD interval test is similar to the sampling-theoretic approach, the concept and results are different. Kim (1991) shows that the HPD interval test for stationarity of regression parameters outperforms in power the conventional non-Bayesian techniques such as the test of Quandt (1958) and the cusum and cusum of squares tests of Brown et al. (1975), even if the diffuse prior is used.

The posterior probability distribution function of τ, $\pi(\tau|R)$, is defined as

$$\pi(\tau|R) = \pi(\tau)\tau^{-1/2}(n-\tau)^{-1/2}\Gamma((\tau-1)/2)\Gamma((n-\tau-1)/2) \times S_1^{-(\tau-1)/2}S_2^{-(n-\tau-1)/2}, \quad (2)$$

where $S_1 = \sum_{t=1}^{\tau}(R_t - \bar{R}_1)^2$, $S_2 = \sum_{t=\tau+1}^{n}(R_t - \bar{R}_2)^2$, $\Gamma(\cdot)$ is a gamma function, and $\pi(\tau)$ is a prior distribution of τ (See in detail Kim (1991) for the derivation of Eqs. (1) and (2)). With no prior information on τ, assigning a uniform prior implies that the term $\pi(\tau)$ has no impact in determining the posterior probability of τ. Note that $F(1)$ is the conditional F-test statistic for the null hypothesis given a change-point τ. Therefore, the unconditional p-value for variance parameter stationarity is the weighted average of the conditional p-values based on an F-test given τ (i.e., the inside bracket in Eq. (1)) with the probability weights determined by the posterior distribution of τ in Eq. (2).

If the unconditional p-value from Eq. (1) is less than an assigned nominal significance, we calculate the posterior probability of each data point using Eq. (2) and select the data point having the highest posterior probability as the change-point for the variance parameter. To avoid any spurious detection of variance parameter shifts from too small a sample, we require at least 10 data points in each regime and 44 data points for the initial test sample. [7] It would be worthwhile, however, that the choice of too many initial test sample has possibility of containing observations from two different regimes. The 1% significance level will be used in the HPD interval test throughout the paper for a conservative detection of variance change-points.

2.2. Evidence of structural change in stock returns

We use the sample of daily returns from January 4, 1965 to December 29, 1995 on the 30 stocks in the Dow Jones Industrial Average, the Standard and Poors 500 index portfolio and the Value-Weighted (VW) index portfolio from the Center for Research in Security Prices (CRSP) at the University of Chicago. Table 1 provides the number of variance change-points detected at the 1 percent significance level by the sequential HPD test in the daily return time-series for each of the 30 stocks and two indexes for a structural change specification. The large number of variance changes in each time series indicates that the sequential mixture of normal distributions model can explain the observed kurtosis in the data (unconditional normal distribution). When the variance parameter changes, we also test if the mean parameter also changes. Simultaneous mean change-points (in parenthesis in Table 1) are very rarely detected.

[7] The choice of 10 and 44 data points is of course arbitrary. When we used slightly different minimum data points and initial subsample size, a little different number of variance change-points were detected. However, the estimation results of the model accommodating conditional heteroscedasticity and structural change are qualitatively almost the same.

Table 1
Comparison of the unconditional normal distribution (UND) and sequential mixture (SM) of normal distributions for daily stock returns: January 4, 1965 to December 29, 1995.
Numbers in parentheses indicate the number of mean changes when variance change has been detected.

I.D. No.	Security (or Index)	Sample size	Number of variance change-points	Excess kurtosis[a]		Ljung–Box Q(12) statistics[b]			
						Standardized residuals		Squared standardized residuals	
				SM	UND	SM	UND	SM	UND
1	Allied Signal	7800	81 (0)	1.103	24.617	41.81	52.70	30.77	1846.83
2	Alcoa	7802	61 (1)	0.573	7.278	162.96	135.35	23.95	830.09
3	American Exp	5821	60 (0)	0.410	7.452	39.94	43.28	21.37	1596.32
4	AT&T	7801	86 (2)	0.802	19.316	25.02	52.67	30.04	2013.96
5	Boeing	7802	70 (0)	0.793	4.683	56.63	23.85	23.21	307.88
6	Caterpillar	7802	83 (3)	0.893	7.453	141.50	154.06	25.66	915.95
7	Chevron	7802	67 (2)	0.698	4.740	84.54	87.72	27.87	1057.57
8	Coca Cola	7802	68 (1)	0.850	15.893	51.68	60.97	29.40	2489.21
9	Disney Walt	7802	67 (4)	1.107	10.672	87.01	90.12	8.18	1034.98
10	Du Pont	7801	75 (2)	0.900	5.425	33.67	25.18	29.74	488.83
11	Eastman Kodak	7802	83 (1)	0.747	23.567	19.36	55.65	19.59	1603.42
12	Exxon	7802	59 (0)	0.682	22.462	74.55	77.73	38.26	2421.98
13	General Electric	7802	69 (1)	0.372	5.579	34.25	52.36	11.11	2541.75
14	General Motors	7801	54 (0)	0.470	7.614	42.99	47.50	8.35	1169.36
15	Goodyear	7802	67 (2)	0.797	12.621	39.68	64.95	20.63	930.66
16	Hewlett-Packard	7802	85 (0)	1.360	4.486	48.94	41.10	14.83	502.46
17	IBM	7799	63 (0)	0.761	12.363	10.88	19.20	18.27	471.22
18	International Paper	7802	78 (0)	0.680	11.657	64.17	92.51	11.62	585.33
19	Johnson & Johnson	7802	85 (0)	0.633	4.985	86.48	98.71	52.69	1324.00
20	McDonalds	7421	51 (2)	0.501	6.437	53.58	56.56	18.54	1327.04
21	Morgan JP	6760	53 (2)	0.947	48.136	151.83	107.07	115.76	1933.30
22	Merck	7802	72 (1)	0.478	3.043	84.20	93.13	19.86	976.76
23	3M	7802	66 (0)	0.708	19.106	69.03	69.70	19.46	519.30
24	Phillip Morris	7801	78 (3)	1.151	8.577	78.90	86.30	13.36	170.61
25	Proctor & Gamble	7802	72 (1)	0.799	34.009	40.13	46.12	14.42	2080.51
26	Sears	7802	81 (3)	0.523	12.750	52.32	66.37	16.63	1342.05
27	Travelers	2318	22 (1)	0.509	10.728	10.50	19.37	13.60	679.77
28	Union Carbide	7802	82 (0)	0.665	7.974	50.92	34.57	33.83	1620.65
29	United Tech.	7800	73 (1)	0.914	3.453	79.59	81.83	26.30	572.59
30	Wal-Mart	5838	59 (1)	0.707	5.029	48.47	84.40	39.72	713.13
31	S&P 500	7802	70 (3)	0.326	42.779	175.87	144.92	22.51	807.62
32	VW CRSP	7802	74 (4)	0.461	31.178	337.03	320.10	18.67	1102.53

[a]The upper and lower 1 percentile points in the distribution of the excess kurtosis statistic are 0.13 and −0.11, respectively.
[b]The upper 1 and 5 percentile points in the χ^2 distribution for the Ljung–Box Q(12) statistic are 26.22 and 21.03, respectively.

The characteristics of the standardized residuals from the sequential mixture model can also provide a basis of comparison. Table 1 also contains the excess kurtosis coefficients of the standardized residuals for the alternative models. The departure from normality for the sequential mixture of normal distributions model is considerably less than the unconditional model for every stock and index. However, the Ljung–Box Q(12) (Ljung and Box, 1978) statistics for the standardized residual of the sequential mixture model are statistically significant (at the one percent level) for 26 of the 30 stocks and both indexes. This evidence indicates significant time dependence in the return series not modeled by the sequential mixture specification. The Ljung–Box Q(12) statistics for the squared standardized residual of the sequential mixture model are also statistically significant (at the one percent level) for 11 of the 30 stocks and the VW index. This evidence indicates that there is also time dependence in the variance process that is not modeled by the sequential mixture specification for some stocks and indexes. Therefore, although the sequential mixture model provides less of a departure from conditional normality for all stocks and indexes, there should be some benefit to combining both empirical regularities (structural change and time dependence) into one model specification.

It would be argued that the presence of one (or more) extreme return observation (outlier) in a regime will significantly affect the test statistic and spuriously detect the change-point even when we are still in the same regime. To examine this argument, we count how many extreme observations are on the detected change-points (event time) of the 30 stocks. We use 4.5% as a pre-set threshold value to determine extreme observations. That is, if daily returns are greater than 4.5% or less than −4.5%, they are presumed to be extreme observations. Each stock has on average 140.3 extreme observations (87.8 are positive extreme observations, and 52.5 are negative extreme observations). Among those, only 4% occur on the detected change-points, while 96% occur on the time-points that are not the change-points. In addition, the average return of the extreme observations occurred on the change-points is not significantly different in magnitude from the average return of the other extreme observations. [8] We therefore argue that the detection of the change-points is not necessarily severely affected by extreme return observations.

In order to examine the information contents of the detected change-points, we also calculate the average returns around the detected change-points over the period [−25, +25]. Note that Day 0 indicates the detected change-point or event date. The average return on the event date is apparently much greater than the average returns on the other dates. For example, the daily average return on the

[8] Among 87.8 positive extreme observations (52.5 negative extreme observations), the average daily return of the extreme observations occurred on the change-points is 6.9% (−7.2%), and that of the extreme observations occurred on the non-change-points is 6.1% (−6.5%).

event date of the CRSP value-weighted market index is 0.294%, while that on the other dates is 0.002%. We obtained similar results for the 30 stocks. It is obvious from the above results that this high return on the event date is not necessarily driven by extreme returns on the event date. These results indicate that the *statistical* procedure has detected the structural shift points that might contain some economic implications.

3. A joint model of time dependence and structural change

Lamoureux and Lastrapes (1990) provide evidence through simulation and actual stock return data that ‘‘ignoring simple structural shifts in unconditional volatility (i.e., model misspecification) can lead to the spurious appearance of extremely strong persistence’’. They estimate a GARCH(1,1) model with dummy variables in the variance equation for arbitrarily chosen subsamples. In particular, they allow for structural shifts every 302 observations in a time series of 4228. The results indicate that the sum of the GARCH parameters for individual stocks decreases substantially when the dummies are included. Lamoureux and Lastrapes (1990) conclude that discrete shifts in unconditional variance are a type of persistence in variance whereby shocks contain no information about future evolution. They also conclude that the next logical step is to develop a means of identifying the timing of such discrete shifts. Note that the sequential mixture of normal distributions model provides this identification.

The Ljung–Box Q(12) statistics in Table 1 indicate that the sequential mixture model applied to individual stocks and indexes contained significant time dependence. Thus, the specification we will use for the conditional mean,

$$m_t = c_0 + \theta h_t + \sum_{j=1}^{l} b_j R_{t-j} + \sum_{j=1}^{n} a_j \epsilon_{t-j}, \qquad (3)$$

contains an autoregressive component of order l, a moving average component of order n, a response coefficient θ to changes in contemporaneous variance.

As a time dependence specification, Glosten et al. (1993) propose a model to capture both volatility clustering and the leverage effect by adding a term for the asymmetry to a GARCH model. The conditional variance specification for the time-dependent model with structural change is

$$h_t = \omega + \beta_1 h_{t-1} + \sum_{j=1}^{3} \alpha_j \epsilon_{t-j}^2 + \gamma_1 S_{t-1}^- \epsilon_{t-1}^2 + \delta X_h, \qquad (4)$$

where $S_{t-1}^- = 1$ if $\epsilon_{t-1} < 0$ and $S_{t-1}^- = 0$ otherwise. This allows the impact of the squared residual on conditional volatility to be different when the lagged 1 residual is negative than when the lagged 1 residual is positive. Therefore, the leverage effect is captured with the hypothesis that $\gamma_1 > 0$. This definition is a GJR(1,3) model specification for conditional variance augmented with an exogenous vector of structural change in the variance, X_h. This vector contains the sample variance for each observation in the time-series from the sequential mixture model. This definition is computationally more efficient than assigning a dummy variable vector for each change point estimated by the sequential mixture model. Now we have a model specification that can accommodate discrete shifts in the constant term of the conditional variance. In order to capture the linear dependence in the variance, we jointly estimate, via maximum likelihood, this GJR(1,3)-MX model with an ARMA(2,1) in the mean. Note that without the exogenous variable, X_h, this model becomes GJR(1,3)-M model.

3.1. Estimation of the joint model

The GJR-MX estimates are presented in Table 2. These estimates provide a different interpretation of the data from those of the GJR-M model. The coefficient of contemporaneous variance, θ, in the conditional mean is still generally positive, but most are insignificant. The constant term in the conditional variance for the GJR-M model is significantly positive for all stocks and indexes. For the GJR-MX model in Table 2, however, the constant term in the conditional variance is significantly positive for only 5 of 30 stocks and no indexes. This is consistent with replacing the constant unconditional variance assumption with a model of structural change. The coefficient of the structural change vector in the conditional variance, δ, is significantly positive for all 30 stocks and both indexes.

In the GJR-M model, the coefficient of lagged conditional variance, β_1 is significantly positive in the range of 0.91 to 0.98 for all stocks and indexes (see also Kim and Kon, 1994). This evidence of strong persistence is no longer supported in the GJR-MX model. In Table 2, β_1 is not significant for all 30 stocks and the indexes. [9] This evidence is consistent with that of Lamoureux and Lastrapes (1990). That is, failure to take account of deterministic structural shifts in a GJR-M model will overstate the persistence in variance. Our result of overstatement of persistence in variance is much greater on actual stock return data than in Lamoureux and Lastrapes (1990), because we identify the timing of the discrete changes in variance (estimation of change points) rather than use arbitrary dummy variables. This result is also consistent with the Day and Lewis (1992) finding of a drop in the persistence parameter for the Standard and Poors 100 index when employing the implied variance from the index option as an exogenous variable.

Glosten et al. (1993) argue that when using the monthly risk-free nominal interest rates and January and October dummy variables as the exogenous vari-

[9] After first estimating the sequential mixture model, we then estimated the GJR-M model with standardized residuals which are returns divided by the corresponding regime's variance. In this estimation, β_1 is also not significant for 27 of the 30 stocks. However, it is still significant for the indexes.

Table 2
GJR(1,3)-MX(returns) estimates
$R_t = m_t + \epsilon_t$, $\epsilon_t \sim$ N(0, h_t), where $m_t = c_0 + \theta h_t + \Sigma_{j=1}^{2} b_j R_{t-j} + a_1 \epsilon_{t-1}$, $h_t = \omega + \beta_1 h_{t-1} + \Sigma_{j=1}^{3} \alpha_j \epsilon_{t-j}^2 + \gamma_1 S_{t-1}^{-} \epsilon_{t-1}^2 + \delta X_h$, and the GJR(1,3)-MX(returns) model is X_h is an exogenous vector that contains the sample variance for each observation in the time series based on the estimated change-points in variance of stock returns. Numbers in parentheses indicate the robust t-statistics of Bollerslev and Wooldridge (1988).

I.D. No.	$\hat{\theta}$	$\hat{\omega}$ ($\times$100)	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\gamma}_1$	$\hat{\delta}$
1	0.4775	0.0005	−0.0082	0.0429	0.0055	−0.0043	0.0032	0.9278
	(1.256)	(1.116)	(−0.125)	(3.302)	(0.807)	(−0.653)	(0.189)	(14.619)
2	0.1187	0.0000	−0.0011	0.0522	0.0199	−0.0054	−0.0109	0.9201
	(0.921)	(0.030)	(−0.078)	(3.419)	(1.893)	(−0.624)	(−0.536)	(34.679)
3	−0.0357	0.0007	0.0340	0.0174	0.0005	−0.0153	0.0070	0.9370
	(−0.234)	(1.074)	(0.409)	(1.123)	(0.060)	(−2.297)	(0.332)	(11.527)
4	1.5232	0.0002	−0.0758	0.0534	−0.0007	−0.0009	−0.0204	1.0070
	(1.668)	(1.638)	(−1.305)	(3.905)	(−0.105)	(−0.146)	(−1.081)	(18.294)
5	1.8126	0.0011	0.0010	0.0366	0.0046	−0.0072	−0.0134	0.9303
	(1.559)	(2.738)	(0.084)	(2.897)	(0.535)	(−1.767)	(−0.711)	(56.915)
6	0.0792	0.0009	0.0012	0.0467	−0.0021	−0.0097	−0.0145	0.9213
	(0.652)	(2.515)	(0.079)	(3.409)	(−0.288)	(−1.573)	(−0.815)	(37.702)
7	0.0570	0.0005	−0.0771	0.0480	0.0171	−0.0057	−0.0366	0.9926
	(0.329)	(1.580)	(−1.327)	(3.423)	(2.130)	(−0.728)	(−2.159)	(17.822)
8	−0.2280	0.0007	0.0001	0.0271	−0.0020	−0.0256	0.0199	0.9426
	(−1.029)	(2.913)	(0.031)	(2.155)	(−0.267)	(−7.491)	(1.080)	(42.198)
9	−0.0878	0.0006	0.0009	0.0258	−0.0025	−0.0245	0.0143	0.9536
	(−0.356)	(1.356)	(0.078)	(1.993)	(−0.578)	(−12.888)	(0.788)	(39.228)
10	2.8184	0.0005	−0.0555	0.0637	0.0025	−0.0128	−0.0332	0.9939
	(2.113)	(1.306)	(−1.452)	(4.241)	(0.279)	(−2.330)	(−1.670)	(29.824)
11	0.6284	0.0008	−0.0023	0.0168	−0.0118	−0.0125	0.0066	0.9680
	(1.459)	(2.382)	(−0.113)	(1.340)	(−1.990)	(−1.619)	(0.379)	(37.054)
12	0.9311	0.0001	−0.1195	0.0412	0.0268	0.0118	−0.0152	1.0296
	(2.053)	(0.404)	(−1.246)	(3.030)	(2.530)	(1.340)	(−0.879)	(11.696)
13	0.2158	0.0007	0.0104	0.0077	−0.0170	−0.0227	0.0082	0.9684
	(0.781)	(1.228)	(0.250)	(0.564)	(−1.956)	(−2.744)	(0.437)	(23.034)
14	0.4018	0.0005	−0.0088	0.0302	−0.0011	−0.0089	−0.0225	0.9679
	(1.154)	(1.627)	(−0.154)	(2.158)	(−0.144)	(−1.122)	(−1.300)	(17.400)
15	1.0700	0.0007	−0.0626	0.0287	0.0115	−0.0124	0.0071	0.9948
	(1.461)	(1.553)	(−1.058)	(2.071)	(1.387)	(−1.600)	(0.446)	(18.226)
16	0.6899	0.0002	0.0253	0.0260	−0.0111	−0.0170	−0.0101	0.9778
	(1.504)	(0.426)	(0.683)	(2.087)	(−1.986)	(−3.196)	(−0.578)	(28.356)
17	−0.1739	0.0004	−0.0779	0.0058	0.0001	−0.0121	0.0199	1.0506
	(−0.200)	(1.249)	(−1.609)	(0.597)	(0.017)	(−5.176)	(1.278)	(20.602)
18	1.0229	0.0007	−0.0387	0.0290	0.0018	−0.0068	−0.0055	0.9913
	(0.921)	(1.833)	(−0.586)	(2.159)	(0.206)	(−0.847)	(−0.302)	(15.549)
19	0.2254	0.0002	−0.0780	0.0258	−0.0010	−0.0097	0.0422	1.0162
	(0.279)	(0.987)	(−1.717)	(1.928)	(−0.138)	(−2.055)	(2.300)	(25.962)
20	−0.0572	0.0009	−0.0329	0.0058	−0.0197	−0.0114	0.0344	1.0006
	(−0.110)	(1.654)	(−0.418)	(0.555)	(−2.313)	(−1.460)	(1.843)	(12.916)

Table 2 (continued)

I.D. No.	$\hat{\theta}$	$\hat{\omega}$ (×100)	$\hat{\beta}_1$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\gamma}_1$	$\hat{\delta}$
21	3.9825	0.0002	−0.0063	0.0708	0.0292	−0.0132	−0.0085	0.8872
	(3.360)	(1.065)	(−0.230)	(5.947)	(3.005)	(−1.770)	(−0.426)	(30.227)
22	0.2513	0.0006	−0.0728	0.0313	−0.0127	−0.0032	−0.0018	1.0144
	(0.959)	(1.573)	(−1.293)	(2.416)	(−1.793)	(−0.467)	(−0.111)	(19.772)
23	0.3205	0.0006	−0.0023	0.0212	−0.0196	−0.0037	0.0114	0.9546
	(1.251)	(1.943)	(−0.117)	(1.557)	(−2.885)	(−0.602)	(0.644)	(29.852)
24	0.6397	0.0006	−0.0020	0.0458	0.0030	0.0159	0.0003	0.9093
	(1.293)	(1.621)	(−0.104)	(4.136)	(0.636)	(2.044)	(0.037)	(34.165)
25	0.5130	0.0006	0.0054	0.0205	−0.0194	−0.0050	0.0107	0.9506
	(1.255)	(2.816)	(0.185)	(1.399)	(−2.408)	(−0.586)	(0.556)	(26.058)
26	1.0384	0.0005	−0.0698	0.0399	−0.0087	−0.0029	−0.0236	1.0320
	(1.613)	(1.787)	(−1.409)	(2.631)	(−1.358)	(−1.039)	(−1.210)	(21.253)
27	0.9551	0.0006	−0.0914	0.0152	−0.0227	0.0090	0.0553	1.0483
	(1.141)	(0.892)	(−1.822)	(0.582)	(−1.800)	(0.440)	(1.434)	(17.715)
28	1.6761	0.0004	−0.0893	0.0457	0.0018	−0.0105	−0.0249	1.0374
	(1.787)	(1.210)	(−1.300)	(3.218)	(0.237)	(−1.781)	(−1.472)	(16.422)
29	1.0746	0.0008	−0.0100	0.0155	0.0063	0.0081	0.0155	0.9377
	(1.331)	(1.816)	(−0.138)	(1.254)	(0.747)	(1.119)	(0.901)	(13.158)
30	0.0040	0.0004	−0.1202	0.0527	0.0069	−0.0102	−0.0030	1.0576
	(0.062)	(0.597)	(−1.784)	(3.733)	(1.012)	(−1.466)	(−0.152)	(17.442)
31	−4.3816	0.0000	0.0933	−0.0145	−0.0107	−0.0139	0.0480	0.9230
	(−2.096)	(0.605)	(0.952)	(−1.491)	(−1.362)	(−3.950)	(3.029)	(9.482)
32	−8.1395	−0.0001	0.0150	−0.0233	0.0007	−0.0151	0.0682	0.9811
	(−3.473)	(−1.089)	(0.255)	(−3.256)	(0.131)	(−2.235)	(4.738)	(15.989)

ables in the conditional variance equation, they find seasonal patterns in volatility and quite low persistence of conditional variance. When we use daily returns on the Standard and Poors index and daily returns on the 30-day commercial paper as the risk-free nominal interest rates, we find that the estimated coefficients on those exogenous variables are also statistically significant. We find, however, that persistence of conditional variance is still highly significant and the magnitude of the corresponding coefficient, β_1, is not noticeably changed. When the vector of the structural change in the variance, X_h, is added into the exogenous variables, the statistical significance of the seasonality dummy variables and the nominal interest rates turns out to be insignificant and persistence of conditional variance is also insignificant.

The coefficients of the first-lag squared residual, α_1, are also reduced in both magnitude and statistical significance in GJR-MX from the GJR-M model. In Table 2, α_1 is significantly positive for only 20 of the 30 stocks and significantly negative for the VW index. Notice that in the GJR-M model, it is significantly positive for 28 stocks and all the indexes. These results indicate that the sequential mixture model takes some ARCH effects for a signal of structural change. There is, however, approximately the same frequency of statistical significance in the coefficients of the second and third lag squared residuals, α_2 and α_3, for both models. Finally, the hypothesis of a leverage effect ($\gamma_1 > 0$) is still supported for the indexes in both direction and statistical significance. For all individual stocks, however, γ_1 is positive for only 15 of the 30 stocks and significantly positive for only 1.

3.2. Diagnostics of the joint model and comparison with other models

The diagnostics of the joint model are contained in Table 3. The Ljung–Box Q(12) statistics for the standardized residuals and squared standardized residuals for the GJR-MX specification indicate that most of the linear dependence in the variance has been captured in this model specification. That is, no Q(12) statistic for any stock or index exceeds the one percent critical value of 26.22.

The excess kurtosis coefficients of the GJR-MX specification in Table 3 are much lower than the unconditional normal distribution model and of the same order of magnitude as the sequential mixture model in Table 1. [10] The skewness coefficients are positive for all stocks and the Standard and Poors index. This is in contrast to the significant negative skewness in both the unconditional distribution and the GJR-M model for the VW index in Kim and Kon (1994). French et al. (1987) attribute the negative skewness in the standardized residuals of the stock indexes to the negative relation between the unexpected component of volatility and the unexpected excess holding period return. Campbell and Hentschel (1992) suggest that this observed negative skewness is due to volatility feedback. That is, good and bad news both increase volatility. Simultaneously, the market demands an increase in risk premia that is attained through a stock price decline. Therefore, the price increase associated with good news is dampened by volatility feedback while a price decrease associated with bad news is amplified by volatility feedback. The insignificant skewness for the VW index in Table 3 indicates that either the volatility feedback argument is not supported by the data or the GJR-MX specification may be adequately incorporating any volatility feedback.

Since the GJR-M model is nested in the GJR-MX specification, we can use a likelihood ratio test (LRT) to determine the additional descriptive ability of the

[10] We also tried the suggestion of Bollerslev (1987) suggestion to use the conditional standardized Student-*t* distribution with the additional degrees of freedom parameter in our GJR-MX specification to model the remaining non-normality. All attempts at convergence failed, however, as the algorithm tried to select extremely high values for the degrees of freedom parameter consistent with normality. The sequential detection procedure in Section 2 provides a computationally convenient approach to detect the major change points in the time-series. Either searching for the sequential sampling assumptions that would maximize the Schwarz (1978) criterion for each stock and index or reducing the significance level for detecting nonstationarity will further reduce the remaining leptokurtosis.

Table 3
GJR(1,3)-MX(returns) diagnostics
The GJR(1,3)-MX(returns) model is $R_t = m_t + \epsilon_t$, $\epsilon_t \sim \mathrm{N}(0, h_t)$, where $m_t = c_0 + \theta\ h_t + \Sigma_{j=1}^{2}\ b_j\ R_{t-j} + a_1\ \epsilon_{t-1}$, $h_t = \omega + \beta_1\ h_{t-1} + \Sigma_{j=1}^{3}\ \alpha_j\ \epsilon_{t-j}^2 + \gamma_1\ S_{t-1}^{-}\ \epsilon_{t-1}^2 + \delta\ X_h$, and X_h is an exogenous vector that contains the sample variance for each observation in the time series based on its own change-points in variance of actual returns. The GJR-M model is the GJR(1,3)-MX(returns) model without X_h, the SM-M model is the sequential mixture model with m_t in the mean equation and $h_t = \omega + \delta X_h$, and the SM model is the sequential mixture of normals. LRT indicates -2 times the log-likelihood ratio of the former to the later model ($\mathrm{LRT}_1 \sim \chi^2(1)$, $\mathrm{LRT}_2 \sim \chi^2(10)$, and $\mathrm{LRT}_3 \sim \chi^2(5)$). $Q(12)$ indicates Ljung and Box (1978) statistic with 12 lags.

I.D. No.	Skewness coefficient[a]	Excess kurtosis	Q(12) standardized residual	Q(12) squared standard residual	Log-likelihood value	LRT_1 GJR-M vs. GJR-MX	LRT_2 SM vs. GJR-MX	LRT_3 SM-M vs. GJR-MX
1	0.223	0.884	11.55	19.60	21473.79	1104.86	51.66	18.86
2	0.168	0.645	13.95	7.22	21354.51	731.24	93.46	22.02
3	0.174	0.460	14.36	22.02	15057.13	611.04	45.67	6.46
4	0.230	0.766	9.76	25.95	24748.01	1248.24	47.50	22.42
5	0.261	0.799	9.53	21.08	19970.02	929.64	99.96	16.86
6	0.125	0.881	24.75	10.59	21705.63	1075.22	147.20	56.72
7	0.097	0.655	3.34	14.24	22519.68	770.76	160.46	58.64
8	0.156	0.827	8.47	15.16	22685.40	812.40	76.42	54.08
9	0.239	1.139	20.94	8.35	20482.27	950.10	155.66	16.36
10	0.199	0.869	6.10	11.16	22800.91	802.14	74.32	29.40
11	0.165	1.152	8.73	17.31	22040.32	1094.84	96.29	24.04
12	0.045	0.656	6.38	14.92	24013.86	679.20	104.24	33.65
13	0.136	0.343	9.56	7.64	23017.29	678.96	44.12	13.76
14	0.170	0.449	5.50	4.97	22405.18	726.08	57.94	4.32
15	0.208	0.767	6.08	11.40	21388.56	811.42	59.56	13.50
16	0.023	0.375	4.38	12.11	19972.45	1003.70	65.32	12.98
17	0.240	0.818	8.14	14.91	22957.17	776.50	113.06	11.74
18	0.171	0.700	7.36	7.96	21697.98	877.74	74.10	46.10
19	0.122	0.564	13.00	19.16	22511.41	897.84	121.40	46.84
20	0.185	0.485	12.78	10.38	19870.57	611.70	55.12	26.18
21	0.190	0.728	23.34	9.98	19656.09	691.22	214.00	75.14
22	0.154	0.466	8.36	9.92	22646.13	701.90	95.20	11.44
23	0.120	0.707	21.55	10.54	23276.91	805.58	63.86	22.54
24	0.034	0.843	12.01	5.36	22055.53	815.78	107.08	31.30
25	0.159	0.806	10.10	9.84	23851.24	880.54	63.08	36.30
26	0.181	0.532	16.63	8.87	22379.08	1016.82	69.82	11.58
27	0.165	0.506	6.49	12.35	5940.47	252.16	27.62	12.32
28	0.224	0.699	4.30	22.46	21747.50	1096.88	37.96	19.26
29	0.162	0.886	6.05	17.56	21231.32	798.66	136.94	9.76
30	0.201	0.633	4.58	13.69	15055.14	709.60	71.58	31.46
31	0.052	0.368	12.91	11.98	27449.71	748.12	119.26	16.26
32	−0.008	0.508	18.13	15.57	28111.61	799.20	341.24	16.88

[a]The upper and lower 1 percentile point in the distribution of the skewness statistic are 0.058 and −0.058, respectively.

structural change vectors. LRT_1 is -2 times the log-likelihood ratio of the GJR-M to GJR-MX models and has a χ^2 distribution with one degree of freedom. Note that all of the LRT_1 values in Table 3 are statistically significant. They range from 252.16 to 1248.24, and hence, substantially exceed the 1 percentile critical value of 6.63.

The sequential mixture model is also nested in the GJR-MX specification. LRT_2 is -2 times the log-likelihood ratio of the sequential mixture to GJR-MX models and has a χ^2 distribution with 10 degrees of freedom. The LRT_2 values for all 30 stocks and both indexes in Table 3 are statistically significant (exceed the 1 percentile critical value of 23.21). The smaller the Q(12) statistics for both the standardized and squared standardized residuals, the lower the LRT_2 values. The reason is that there is little time dependence left to model for stocks having smaller Q(12) statistics.

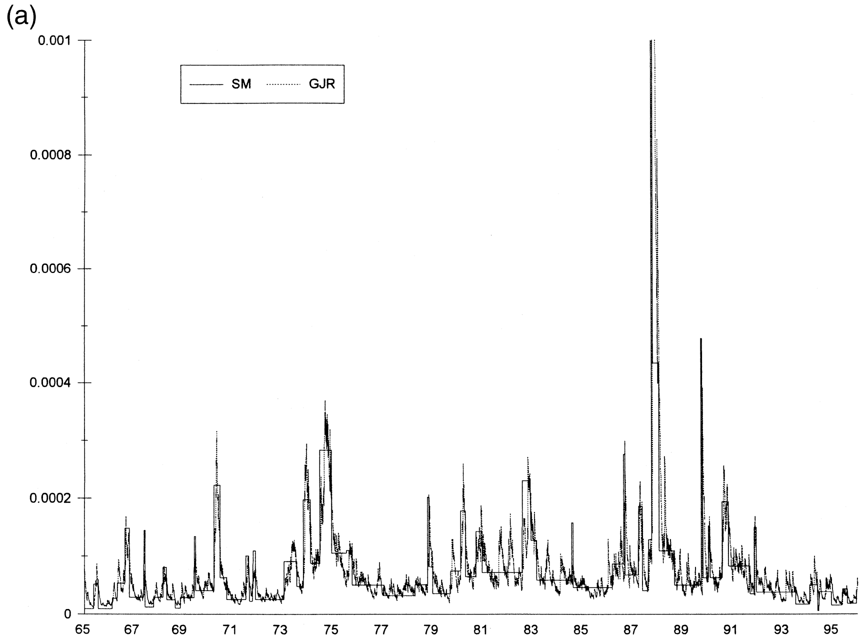
Furthermore, most of the significance among the LRT_2 values is due to the time dependence in the conditional mean. We can see this by testing for the incremental explanatory value of the time dependence in the variance. LRT_3 is -2 times the log-likelihood ratio of the SM-M to GJR-MX models and has a χ^2 distribution with 5 degrees of freedom. The SM-M model is the sequential mixture model with time dependence in the conditional mean (Eq. (3)) and

$$h_t = \omega + \delta X_h \tag{5}$$

for the conditional variance. In Table 3, the LRT_3 values for 10 of the 30 stocks are not statistically significant (less than the 1 percentile critical value of 15.09).

Fig. 1A contains time-series plots of the daily conditional variance estimates from the GJR-M and sequential mixture (SM) models for the Standard and Poors 500 index (ID no. 31) over the whole period from January 4, 1965 to December 29, 1995. Since the GJR-M model assumes that a given realization contains information about future evolution, there is a tendency (relative to the SM model) for the GJR variance estimates to delay and then overshoot variance increases and to be slow to adjust to variance declines. The slow variance decay is particularly evident for the market crash periods of October, 1987 and the mini-crash of October, 1989. Both models detect a sharp increase in volatility. The adjustment process for variance declines is the crucial difference between the two models. The GJR (or GARCH) specification does not accommodate a realization that contains information about a rapid reduction in variance. That is, the GJR (or GARCH) model does not distinguish between a positive residual that is associated with a variance increase from a positive residual that is associated with a variance reduction (lower risk premium). Its persistence parameterization only allows for a slow decay rather than a quick resolution of uncertainty. These features of both models appear more obvious in the scaled-up plot over the sub-period from January 2, 1986 to December 31, 1992 (Fig. 1B).

Fig. 2A provides a comparison of conditional volatility estimates from the GJR(1,3)-MX and the sequential mixture models for the Standard and Poors 500



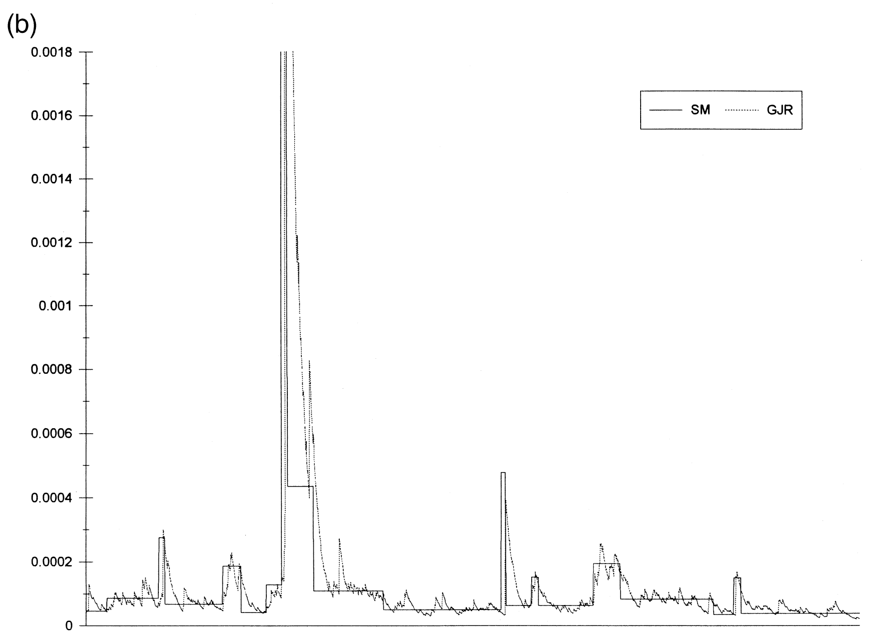


Fig. 1. (a-b) Sequential mixture (SM) vs. GJR variances for the S&P 500.

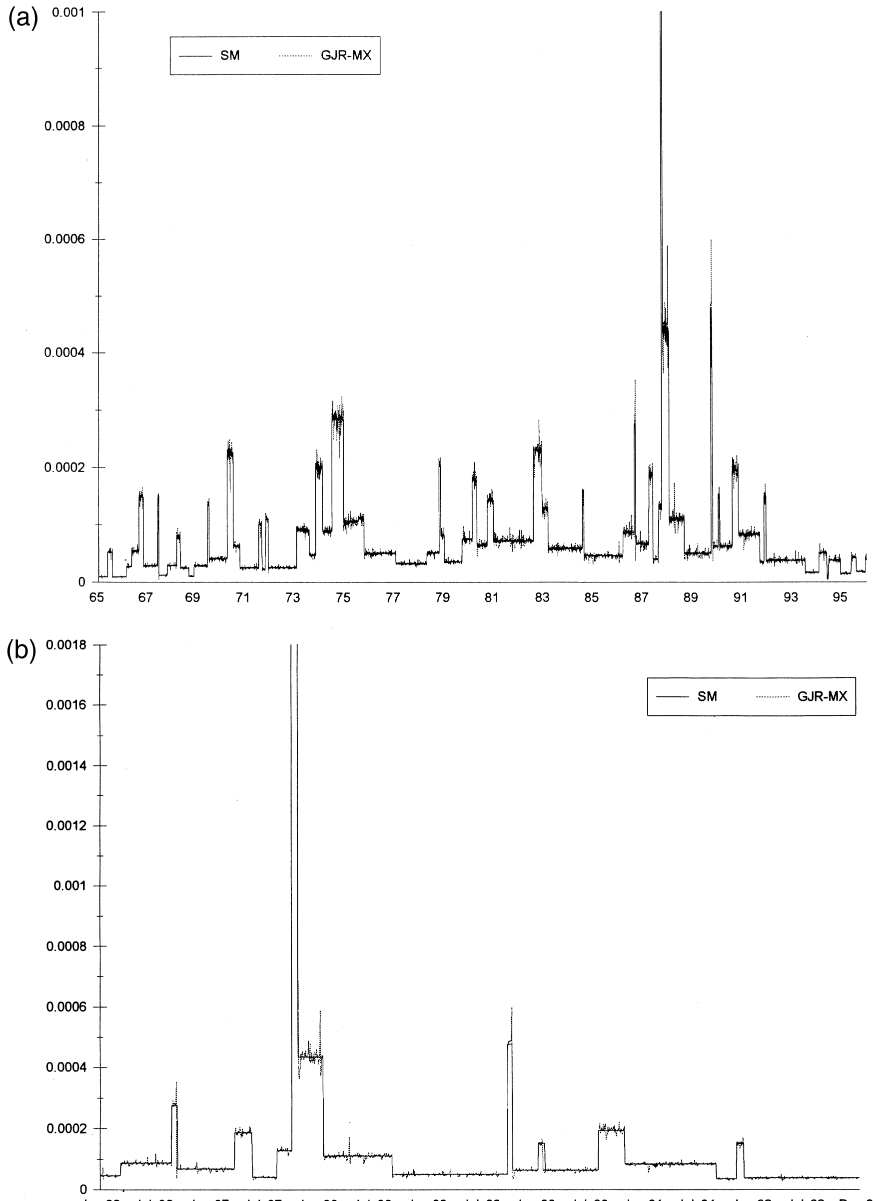


Fig. 2. (a-b) Sequential mixture (SM) vs. GJR-MX variances for the S&P 500.

index. The GJR-MX model follows very closely the pattern of the structural change estimated by the sequential mixture model. The small deviations are associated with the additional fit provided by the significantly negative coefficient of first lagged squared residuals, α_1, and the significantly positive coefficient of the leverage effect, γ_1. Fig. 2B also provides this comparison for the scaled-up sub-period from January 2, 1986 to December 31, 1992.

3.3. Substitutability between a structural change model and a time-dependence model

Since both the sequential mixture and GJR-MX models detected the major volatility increases in the time-series, there is a certain amount of substitutability. In the above GJR-MX procedure, we estimated the sequential mixture model change points on the return series to construct the exogenous vectors. An alternative is to estimate the sequential mixture model change points on the standardized residual series from the GJR-M model. [11] At a minimum, this procedure represents a diagnostic for the GJR-M specification. [12] From the estimated change points on the residuals, we construct the exogenous vectors and estimate, via maximum likelihood, the GJR-MX(residuals) model specification with Eqs. (3) and (4).

The diagnostics are contained in Table 4. The Ljung–Box Q(12) statistics for the standardized residuals and squared standardized residuals indicate that most of the linear dependence in the mean and variance has been captured in this model specification as well. That is, only one stock (ID no. 8) had a Q(12) statistic for the squared standardized residuals that exceeds the one percent critical value of 26.22. The skewness and excess kurtosis coefficients of the GJR-MX(residuals) specification in Table 4 exhibit some more negative skewness and leptokurtosis than the GJR-MX(returns) model in Table 3, especially, for the indexes.

Note that the GJR-M model is nested in the GJR-MX(residuals) specification and we can use an LRT to determine the additional descriptive ability of the structural change vectors. LRT_4 is -2 times the log-likelihood ratio of the GJR-M to GJR-MX(residuals) models and has a χ^2 distribution with one degree of freedom. All of the LRT_4 values in Table 4 are statistically significant. They range from 37.36 to 593.56, and hence, substantially exceed the 1 percentile critical value of 6.63. This is clear diagnostic evidence that the GJR-M model alone is misspecified. Thus, regardless of the order of introducing structural change into the GJR-M specification, its contribution is significant.

Since the only difference between the two GJR-MX models, GJR-MX(returns) and GJR-MX(residuals), is the order of the application of the estimation of the

[11] The number of change-points in variance on the standardized residual series is on average 39 for the 30 stocks, while that on the actual return series is on average 70. These results indicate that a straightforward estimation of the GJR-M model (or any (G)ARCH model) tends to somewhat destroy the impact of breaks.

[12] We are grateful to Victor Ng for this suggestion.

Table 4
GJR(1,3)-MX(residuals) diagnostics
The GJR(1,3)-MX(residuals) model is the same as the GJR(1,3)-MX(returns) model except that the exogenous vector X_h contains the sample variance for each observation in the time series based on the change-points in variance of the standardized residual series from the the GJR-M model. The GJR-M model is the GJR(1,3)-MX(returns) model without the exogenous variable. LRT_4 indicates -2 times the log-likelihood ratio of the GJR-M to the GJR-MX(residuals) model ($LRT_4 \sim \chi^2(1)$). Log-odds is the log-likelihood ratio of the two models having the same number of parameters estimated. $Q(12)$ indicates Ljung and Box (1978) statistic with 12 lags.

I.D. No.	Skewness coefficient	Excess kurtosis	Q(12) standardized residual	Q(12) squared standard residual	Log-likelihood value	LRT_4 GJR-M vs. GJR-MX (residuals)	Log-odds GJR-MX (residuals) vs GJR-MX (returns)
1	0.059	3.048	8.92	17.72	21109.68	373.13	−365.86
2	0.019	2.465	10.90	16.06	21040.07	99.62	−315.81
3	0.095	1.379	12.38	13.60	14788.80	78.16	−266.44
4	0.106	2.059	11.62	10.36	24420.69	593.60	−327.24
5	0.240	1.273	10.05	4.86	19766.92	520.56	−204.54
6	−0.111	2.573	23.21	5.50	21310.94	279.98	−397.62
7	0.049	2.021	7.27	15.78	22188.83	116.18	−327.29
8	0.082	2.140	6.69	26.74	22296.78	37.36	−387.52
9	0.020	4.015	15.88	4.28	20060.86	167.52	−391.29
10	0.196	1.640	9.62	13.58	22457.92	121.78	−340.18
11	0.040	2.282	14.41	9.88	21686.59	395.80	−349.52
12	0.019	1.124	4.18	9.47	23778.01	215.50	−231.85
13	0.117	1.072	10.94	10.80	22706.50	66.50	−306.23
14	0.192	2.183	8.86	7.04	22117.74	149.14	−288.47
15	0.150	1.801	10.28	13.03	21091.57	222.40	−294.51
16	−0.049	2.021	6.14	3.88	19594.40	255.00	−374.35
17	0.064	3.074	9.31	7.99	22573.27	106.12	−335.19
18	0.116	1.468	7.55	14.59	21428.21	337.72	−270.01
19	0.001	1.258	13.45	20.81	22163.26	210.02	−343.91
20	0.070	1.429	3.05	18.05	19594.50	65.66	−273.02
21	0.016	2.968	21.69	7.78	19427.29	239.58	−225.82
22	0.056	1.468	12.21	13.59	22316.33	47.28	−327.31
23	−0.009	0.707	21.55	10.54	23276.91	325.14	−240.22
24	−0.330	7.428	8.74	2.14	21780.09	292.02	−270.88
25	0.154	1.431	10.20	19.18	23530.03	240.14	−320.20
26	0.123	1.747	12.45	6.84	22007.18	271.86	−372.48
27	−0.085	2.464	5.52	8.71	5822.66	41.04	−105.81
28	0.274	1.740	6.51	11.48	21433.14	428.36	−334.26
29	0.155	1.648	6.29	10.69	20922.54	223.56	−287.55
30	0.029	1.676	6.06	9.42	14792.21	187.42	−261.09
31	−0.258	2.860	7.73	6.29	27115.54	84.86	−331.63
32	−0.331	3.036	13.91	6.67	27733.82	45.18	−377.01

change point vector, we can compare the procedures on the basis of log-odds. The last column of Table 4 indicates that the log of the ratio of the GJR-MX(residuals) likelihood to the GJR-MX(returns) likelihood favors the GJR-MX(returns) estimation method for all stocks and indexes. This result is consistent with the lower excess kurtosis statistics for the standardized residuals of the GJR-MX(returns) estimation procedure.

4. Common features in structural changes

Structural changes are caused by relevant economic news, market-wide or firm-specific. The interesting question is whether structural changes in the variance that are detected in individual stocks are actually shared in common. According to Engle and Kozicki (1993), it could be said that if two sets of stock i's variances, one estimated from its own change-points (X_{h_i}) and the other estimated from the change-points of the market index m (X_{h_m}), have a common feature (i.e., a sharing of variance change-points with the market and across individual stocks), then $X_{h_i} - \lambda_i X_{h_m}$ has not the common feature, where λ_i is a non-zero constant. Therefore, if there is a sharing of variance change-points, the differenced vector $X_{h_i} - \lambda_i X_{h_m}$ should not be significant when it is used as an exogenous variable in the variance equation (GJR-MX(difference)) instead of X_{h_i}, and persistence of conditional variance should still be prominent. Since the shared variance change-points offset each other, the characteristics of capturing persistence of conditional variance would disappear in the differenced exogenous variable. We use two different values for the constant λ_i; one is the systematic risk β_i, and the other is one. Since the overall results are qualitatively the same, we report the results only for the case where the systematic risk is used. The Standard and Poors 500 index is used as a proxy for the market index.

Table 5 presents the diagnostics of the GJR-MX(difference) model and provides strong evidence that there is the common feature of sharing the structural changes in the variance. The magnitude of the coefficient (δ_{i-m}) on the differenced exogenous variable $X_{h_i} - \beta_i X_{h_m}$ is quite small in comparison with the coefficient on the single exogenous vector X_{h_i}. Furthermore, 17 of the 30 stocks have insignificant coefficients. Even though the coefficient estimates of the other 13 stocks are statistically significant, their magnitude is relatively small. The likelihood ratio test also provides similar results. The coefficient associated with volatility persistence, β_1, of the GJR-M model is very significantly positive. Even after the differenced exogenous variable is added into the GJR-M model as an exogenous variable, the magnitude and the statistical significance of β_1 are almost the same, even for the stocks whose estimates of the coefficient δ_{i-m} are statistically significant.

Another approach to test the sharing of variance change-points is to add X_{h_m} into the previous GJR-MX model (GJR-MX(own)) as the second independent exogenous variable and to examine whether this addition increases the descriptive ability of the model. If there is the common feature, the increase of the descriptive

Table 5
GJR(1,3)-MX(difference) diagnostics
The GJR(1,3) − MX(difference) model is the same as the GJR(1,3) − MX(returns) model except for the exogenous variable. That is, $R_t = m_t + \epsilon_t$, $\epsilon_t \sim$ N(0, h_t), where $m_t = c_0 + \theta\ h_t + \Sigma_{j=1}^{2}\ b_j\ R_{t-j} + a_1\ \epsilon_{t-1}$, $h_t = \omega + \beta_1\ h_{t-1} + \Sigma_{j=1}^{3}\ \alpha_j\ \epsilon_{t-j}^2 + \gamma_1\ S_{t-1}^{-}\ \epsilon_{t-1}^2 + \delta_{i-m}\ X_{h_{i-m}}$. The exogenous variable $X_{h_{i-m}} = X_{h_i} - \beta_i X_{h_m}$ is a difference vector, where X_{h_i} and X_{h_m} contain the sample variance for each observation in the time series from its own variance change-points and from the variance change-points of the market index returns m, respectively, and β_i is the systematic risk. The GJR-M model is the GJR(1,3) − MX(returns) model without the exogenous variable. LRT_5 indicates −2 times the log-likelihood ratio of the GJR-M to the GJR-MX(difference) model ($LRT_5 \sim \chi^2(1)$).

I.D. No.	GJR-M $\hat{\beta}_1$ (t-stat)	GJR-MX (difference)		LRT_5 GJR-M vs. GJR-MX (difference)
		$\hat{\beta}_1$ (t-stat)	$\hat{\delta}_{i-m}$ (t-stat)	
1	0.8715 (142.22)	0.8513 (108.94)	0.0006 (0.89)	5.92
2	0.9426 (306.62)	0.9335 (217.84)	0.0090 (8.31)	19.06
3	0.9402 (252.33)	0.9423 (235.83)	−0.0048 (−4.00)	12.28
4	0.9575 (654.84)	0.9642 (723.09)	0.0006 (0.71)	6.40
5	0.9547 (341.29)	0.9564 (331.63)	0.0003 (0.65)	0.90
6	0.9442 (302.14)	0.9391 (266.51)	0.0135 (13.41)	71.20
7	0.9417 (277.50)	0.9392 (279.51)	0.0087 (5.32)	15.48
8	0.9366 (270.38)	0.9301 (232.81)	−0.0002 (−0.24)	1.26
9	0.9309 (294.31)	0.9221 (250.42)	0.0011 (4.09)	14.22
10	0.9297 (249.38)	0.9417 (291.05)	−0.0003 (−0.39)	3.54
11	0.8881 (137.42)	0.8429 (94.16)	0.0017 (1.44)	4.00
12	0.8794 (116.70)	0.8770 (109.77)	0.0007 (0.46)	1.42
13	0.9361 (247.32)	0.9428 (270.18)	0.0004 (0.35)	1.54
14	0.9454 (298.42)	0.9482 (321.09)	0.0009 (1.58)	3.10
15	0.9345 (244.49)	0.9295 (215.42)	0.0020 (1.12)	5.14
16	0.9148 (197.08)	0.9015 (153.17)	0.0021 (1.50)	1.64
17	0.9357 (274.96)	0.9446 (265.63)	−0.0011 (−2.33)	2.92
18	0.9328 (247.02)	0.9468 (283.60)	0.0013 (1.31)	6.20
19	0.9031 (158.75)	0.8936 (142.45)	0.0010 (1.00)	1.84
20	0.9431 (266.29)	0.9461 (281.08)	0.0002 (0.36)	1.62
21	0.9210 (183.04)	0.9290 (180.70)	0.0029 (3.67)	9.06
22	0.9166 (162.11)	0.9261 (174.76)	0.0121 (5.40)	20.20
23	0.9135 (192.38)	0.9182 (203.28)	0.0092 (7.51)	25.12
24	0.8981 (233.74)	0.9018 (235.15)	0.0025 (1.30)	7.26
25	0.9102 (200.09)	0.9142 (193.26)	0.0237 (12.36)	59.92
26	0.9098 (218.44)	0.9028 (189.56)	0.0080 (3.94)	7.70
27	0.8893 (70.44)	0.8830 (67.25)	−0.0001 (−0.12)	1.74
28	0.9125 (193.24)	0.9126 (180.76)	0.0132 (14.22)	54.58
29	0.9322 (218.32)	0.9280 (203.48)	0.0170 (8.54)	21.22
30	0.9209 (206.12)	0.9215 (200.27)	0.0006 (0.54)	2.24

ability after the addition of X_{h_m} would not be substantial. Table 6 shows the estimation results of the GJR-MX(SP500) model (with X_{h_m} as a single exogenous variable) and the GJR-MX(2) model (with X_{h_i} and X_{h_m} as two exogenous

Table 6
Comparison among GJR(1,3)-MX models with difference exogenous variables

	GJR-MX(SP500)		GJR-MX(2)			LRT_6 GJR-MX(own) vs. GJR-MX(2)	LRT_7 GJR-M vs. GJR-MX(SP500)	Log-odds GJR-MX(SP500) vs. GJR-MX (own)
	$\hat{\beta}_1$	$\hat{\delta}_m$	$\hat{\beta}_1$	$\hat{\delta}_i$	$\hat{\delta}_m$			
1	0.308 (5.55)	0.586 (10.97)	−0.085 (−2.02)	0.962 (19.80)	0.122 (3.70)	14.08	568.68	−268.09
2	0.454 (5.82)	0.480 (6.50)	−0.092 (−1.07)	0.925 (11.66)	0.205 (4.96)	33.18	399.96	−165.64
3	0.332 (3.22)	0.603 (5.92)	−0.020 (−0.22)	0.916 (10.48)	0.092 (2.05)	67.16	303.00	−154.02
4	0.423 (5.35)	0.505 (6.85)	−0.084 (−1.92)	1.026 (22.59)	0.003 (0.19)	0.70	604.64	−321.72
5	0.317 (2.97)	0.632 (6.07)	−0.036 (−0.63)	0.939 (17.32)	0.073 (2.83)	15.26	436.64	−246.50
6	0.082 (0.89)	0.858 (8.82)	−0.058 (−1.90)	0.926 (24.27)	0.119 (3.78)	21.64	503.52	−285.85
7	0.212 (2.45)	0.724 (8.33)	−0.178 (−3.66)	0.994 (22.51)	0.209 (4.63)	36.60	381.98	−194.39
8	0.226 (2.42)	0.712 (7.46)	−0.054 (−1.80)	0.896 (24.03)	0.112 (2.77)	9.84	457.88	−177.26
9	0.195 (2.39)	0.741 (9.43)	−0.083 (−1.69)	0.930 (16.98)	0.186 (4.75)	26.18	511.70	−219.20
10	0.086 (0.79)	0.851 (7.73)	−0.084 (−1.34)	0.925 (16.82)	0.138 (3.99)	34.72	430.98	−185.58
11	0.555 (12.62)	0.394 (8.94)	−0.013 (−0.40)	0.919 (25.50)	0.082 (2.71)	11.52	516.94	−288.95
12	0.542 (8.00)	0.404 (6.16)	−0.083 (−2.41)	0.935 (22.32)	0.199 (4.89)	28.86	410.36	−134.42
13	0.470 (3.98)	0.498 (4.29)	−0.106 (−1.48)	0.953 (16.72)	0.187 (3.62)	26.20	370.12	−154.42
14	0.421 (3.49)	0.547 (4.60)	−0.068 (−1.37)	0.918 (16.52)	0.142 (3.28)	18.06	510.62	−107.73
15	0.508 (5.93)	0.442 (5.41)	−0.091 (−1.35)	0.999 (16.10)	0.129 (3.29)	12.92	411.60	−199.91
16	0.495 (7.16)	0.468 (6.71)	−0.044 (−1.08)	0.904 (25.36)	0.162 (4.22)	24.08	480.66	−261.52
17	0.281 (2.57)	0.696 (6.26)	−0.179 (−2.22)	0.959 (13.10)	0.242 (5.92)	40.66	452.70	−161.40
18	0.265 (2.56)	0.694 (6.66)	−0.089 (−2.28)	0.946 (24.97)	0.205 (5.52)	29.72	478.48	−199.63
19	0.507 (8.66)	0.385 (7.07)	−0.089 (−5.31)	1.006 (34.72)	0.031 (1.67)	11.30	299.96	−298.94
20	0.071 (0.64)	0.880 (7.83)	−0.187 (−2.36)	1.002 (15.11)	0.164 (2.92)	26.26	282.60	−164.55
21	0.274 (3.81)	0.609 (8.95)	0.002 (0.12)	0.751 (21.56)	0.214 (5.90)	33.76	434.56	−128.33
22	0.242 (2.25)	0.700 (6.43)	−0.149 (−2.89)	0.925 (18.69)	0.261 (5.11)	45.38	366.38	−167.76
23	0.013 (0.16)	0.946 (10.52)	−0.173 (−3.62)	0.921 (20.20)	0.341 (6.71)	38.14	494.38	−155.60
24	0.526 (7.66)	0.422 (6.41)	−0.135 (−4.91)	0.973 (41.91)	0.183 (6.57)	16.02	393.86	−219.96
25	0.072 (0.49)	0.883 (6.00)	−0.093 (−2.97)	0.995 (35.43)	0.096 (2.87)	3.88	455.14	−212.70
26	0.581 (11.30)	0.369 (7.41)	−0.084 (−2.00)	0.948 (22.91)	0.148 (4.42)	26.94	421.78	−297.52
27	0.489 (4.10)	0.431 (3.75)	−0.093 (−1.80)	1.022 (17.67)	0.099 (1.53)	23.58	145.16	−53.75
28	0.469 (7.77)	0.449 (8.05)	−0.083 (−3.34)	0.968 (30.98)	0.104 (3.63)	11.32	488.34	−304.27
29	0.483 (6.22)	0.457 (6.21)	−0.138 (−4.21)	1.013 (33.09)	0.119 (3.28)	15.06	347.28	−225.69
30	0.670 (16.02)	0.244 (6.77)	−0.126 (−3.13)	1.004 (22.81)	0.074 (2.41)	6.46	220.26	−244.67

variables) for each of the 30 stocks. These results show that even though X_{h_m} is used as a unique exogenous variable in the variance equation for individual stocks, persistence of conditional variance decreases substantially and all the coefficients on the variable, δ_m, are statistically significant as we found with X_{h_i} in Table 2. Notice, however, that the decrease of persistence of conditional volatility and the statistical significance of the estimated δ_m are slightly less than when X_{h_i} is used as a unique exogenous variable. LRT_7 also supports the significant role of the information about the variance change-points of individual stocks obtained from the market index. In the GJR-MX(2) model, the addition of X_{h_m} into the GJR-MX(own) model does not significantly alter the overall estimation results of the GJR-MX(own). Most of the estimated coefficients on X_{h_m}, $\hat{\delta}_m$, are significantly positive, but substantially smaller than when X_{h_m} is the unique exogenous variable. The reason is that the common feature is shared or absorbed by the other exogenous variable X_{h_i}. If there is no common feature, the estimated coefficients on X_{h_m} in the GJR-MX(SP500) and the GJR-MX(2) models should not be significantly different. Based on the above results, therefore, we could argue that there is a common feature of sharing variance change-points across individual stocks.

5. Summary and implications

In this paper, we provide evidence that the time-series properties of stock returns include both structural change and time dependence in the conditional mean and conditional variance. The absence of a structural change component tends to overstate the persistence parameter in the GJR-M model specification. More importantly, the GJR-M specification does not distinguish between positive residuals that increase volatility from those that represent a resolution of uncertainty. Since the sequential mixture model of structural change estimates discrete change points in the time-series of volatility in either direction, it provides a diagnostic test of the volatility persistence in the GJR-M specification. A GJR-M model with the sequential mixture model structural change points as an exogenous

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Note to Table 6:
The above GJR-MX models are the same as the GJR(1,3)-MX(returns) model except that the GJR-MX(own) and GJR-MX(SP500) models have one exogenous variable containing the sample variance for each observation in the time series from its own variance change-points (X_{h_i}) and from the variance change-points of the Standard and Poors 500 index (X_{h_m}), respectively, and the GJR-MX(2) has the two exogenous variables (X_{h_i} and X_{h_m}). $\hat{\delta}_i$ and $\hat{\delta}_m$ are the estimated coefficients of the exogenous variables X_{h_i} and X_{h_m}, respectively. The GJR-M model is the GJR(1,3)-MX(returns) model without the exogenous variable. LRT indicates -2 times the log-likelihood ratio of the former to the later model (LRT_6 and $LRT_7 \sim \chi^2(1)$). Log-Odds is the log-likelihood ratio of the two models having the same number of parameters estimated. Numbers in parentheses indicate Bollerslev and Wooldridge's (Bollerslev and Wooldridge, 1988) robust t-statistics.

variable, GJR-MX, indicates that there are significant benefits to incorporating time dependence and structural change in the model specification of the conditional mean and conditional variance of stock returns.

These results have important implications for models that forecast future volatility, for example, in option pricing. The time dependent component of conditional volatility indicates that shocks contain information about the future evolution of volatility (i.e., as in mean reversion). The structural change component assumes that shocks are independent events with no predictive value beyond the recognition of being in a new regime. The GJR-MX model provides the functional form and relative weight of each component.

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